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Bounds for preperiodic points for maps with good reduction

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ABSTRACT

Let K be a number field and let ϕ in $K(z)$ be a rational function of degree $d \geq 2$. Let S be the set of places of bad reduction for ϕ (including the archimedean places). Let $\mathrm{Per}(\phi, K)$, $\mathrm{PrePer}(\phi, K)$, and $\mathrm{Tail}(\phi, K)$ be the set of K-rational periodic, preperiodic, and purely preperiodic points of ϕ, respectively. The present paper presents two main results. The first result is a bound for $|\mathrm{PrePer}(\phi, K)|$ in terms of the number of places of bad reduction $|S|$ and the degree d of the rational function ϕ. This bound significantly improves a previous bound given by J. Canci and L. Paladino. For the second result, assuming that $|\mathrm{Per}(\phi, K)| \geq 4$ (resp. $|\mathrm{Tail}(\phi, K)| \geq 3$), we prove bounds for $|\mathrm{Tail}(\phi, K)|$ (resp. $|\mathrm{Per}(\phi, K)|$) that depend only on the number of places of bad reduction $|S|$ (and not on the degree d). We show that the hypotheses of this result are sharp, giving counterexamples to any possible result of this form when $|\mathrm{Per}(\phi, K)| < 4$ (resp. $|\mathrm{Tail}(\phi, K)| < 3$).

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1. Introduction

Let K be a number field and let $\phi \in K(z)$ be a rational function. Let ϕ^n denote the nth iterate of ϕ under composition and ϕ^0 the identity map. The *orbit* of $P \in \mathbb{P}^1(K)$

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under ϕ is the set $O_\phi(P) = \{\phi^n(P) : n \geq 0\}$. A point $P \in \mathbb{P}^1(K)$ is called *periodic* under ϕ if there is an integer $n > 0$ such that $\phi^n(P) = P$. It is called *preperiodic* under ϕ if there is an integer $m \geq 0$ such that $\phi^m(P)$ is periodic. A point that is preperiodic but not periodic is called a *tail* point. Let $\mathrm{Tail}(\phi, K)$, $\mathrm{Per}(\phi, K)$ and $\mathrm{PrePer}(\phi, K)$ be the sets of K-rational tail, periodic and preperiodic points of ϕ, respectively.

For any morphism $\phi : \mathbb{P}^N \to \mathbb{P}^N$ of degree $d \geq 2$, Northcott [15] proved in 1950 that the total number of K-rational preperiodic points of ϕ is finite. In fact, from Northcott's proof, an explicit bound can be found in terms of the coefficients of ϕ. In 1994, Morton and Silverman [13] conjectured that $|\mathrm{PrePer}(\phi, K)|$ can be bounded in terms of only a few basic parameters.

Conjecture 1.1 *(Uniform boundedness conjecture). Let K be a number field with $[K : \mathbb{Q}] = D$, and let ϕ be an endomorphism of $\mathbb{P}^N$, defined over K. Let $d \geq 2$ be the degree of ϕ. Then there is $C = C(D, N, d)$ such that ϕ has at most C preperiodic points in $\mathbb{P}^N(K)$.*

The conjecture seems extremely difficult to prove even in the simpler case when $(K, N, d) = (\mathbb{Q}, 1, 2)$. Further, in this case, explicit conjectures have been formulated. For instance, Poonen [16] conjectured an explicit bound when ϕ is a quadratic polynomial map over $\mathbb{Q}$. Since every such quadratic polynomial map is conjugate to a polynomial of the form $\phi_c(z) = z^2 + c$ with $c \in \mathbb{Q}$ we can state Poonen's conjecture as follows: Let $\phi_c \in \mathbb{Q}[z]$ be a polynomial of degree 2 of the form $\phi_c(z) = z^2 + c$ with $c \in \mathbb{Q}$. Then $|\mathrm{PrePer}(\phi_c, \mathbb{Q})| \leq 9$. B. Hutz and P. Ingram [11] have shown that Poonen's conjecture holds when the numerator and denominator of c don't exceed 10^8.

This work has two main contributions. The first result gives a bound for $|\mathrm{PrePer}(\phi, K)|$ in terms of the number of places of bad reduction $|S|$ and the degree d of the rational function ϕ. This bound significantly improves a previous bound given by J. Canci and L. Paladino [7].

In the second result, assuming that $|\mathrm{Per}(\phi, K)| \geq 4$ (resp. $|\mathrm{Tail}(\phi, K)| \geq 3$), we prove bounds for $|\mathrm{Tail}(\phi, K)|$ (resp. $|\mathrm{Per}(\phi, K)|$) that depend only on the number of places of bad reduction $|S|$ and $[K : \mathbb{Q}]$ (and not on the degree d). We show that the hypotheses of this result are sharp, Example 5.2 and Example 5.1 give counterexamples to any possible result of this form when $|\mathrm{Per}(\phi, K)| < 4$ (resp. $|\mathrm{Tail}(\phi, K)| < 3$).

Theorem 1.2. *Let K be a number field and S a finite set of places of K containing all the archimedean ones. Let ϕ be an endomorphism of $\mathbb{P}^1$, defined over K, and $d \geq 2$ the degree of ϕ. Assume ϕ has good reduction outside S.*

(a) *If there are at least three K-rational tail points of ϕ then*

$$|\mathrm{Per}(\phi, K)| \leq 2^{16|S|} + 3.$$

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