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Abstract

In a series of papers C. Dartyge and A. Sárközy (partly with other coauthors) studied pseudorandom measures of subsets. In this paper we extend the theory of C. Dartyge and A. Sárközy to several dimensions. We introduce measure for multi-dimensional pseudorandom subsets, and study the connection between measures of different orders. Large families of multi-dimensional pseudorandom subsets are given by using the squares in $\mathbb{F}_q$.

Keywords pseudorandom measure; multi-dimensional subset; finite field.

AMS subject classfications (2010) 11K38; 11K45; 11L40; 11Z05.

§1. Introduction

Numerous papers have been written on pseudorandom measures of subsets in the last decades. In these papers a wide range of goals, approaches, tools is presented. For example, F. R. K. Chung and R. L. Graham [2] proved that there is a surprisingly large class of subset properties, all shared by quasirandom subsets, which are equivalent in the following sense: If a family of subsets satisfies some property in $\Im$, then it must satisfy all the properties in $\Im$. K. F. Roth [12] proved Szemerédi's theorem in the special case, using exponential sums and pseudorandom subsets. W. T. Gowers [7] defined stronger pseudorandomness on subsets, and gave a new proof for general Szemerédi's theorem by using Fourier-analytic methods.

In a series of papers C. Dartyge and A. Sárközy (partly with other coauthors) studied pseudorandom measures of subsets. For details, let $\mathcal{R} \subset \{1, 2, \cdots, N\}$ and define the sequence

$$E_N = E_N(\mathcal{R}) = \{e_1, e_2, \cdots, e_N\} \in \left\{1 - \frac{|\mathcal{R}|}{N},\ -\frac{|\mathcal{R}|}{N}\right\}^N$$

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