Finite multiple zeta values associated with 2-colored rooted trees

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FINITE MULTIPLE ZETA VALUES ASSOCIATED WITH 2-COLORED ROOTED TREES

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Abstract. We define finite multiple zeta values (FMZVs) associated with some combinatorial objects, which we call 2-colored rooted trees, and prove that FMZVs associated with 2-colored rooted trees satisfying certain mild assumptions can be written explicitly as $\mathbb{Z}$-linear combinations of the usual FMZVs. Our result can be regarded as a generalization of Kamano's recent work [K] on finite Mordell-Tornheim multiple zeta values. As an application, we will give a new proof of the shuffle relation of FMZVs, which was first proved by Kaneko and Zagier.

Contents

1. Introduction

In the 1990's, Hoffman [H2] and Zhao [Z] had started independently a theory of mod p multiple harmonic sums. Recently, Kaneko and Zagier introduced a new "adelic" framework to describe the work of Hoffman and Zhao. That is, they defined the $\mathbb{Q}$-algebra $\mathcal{A} := \left(\prod_p \mathbb{F}_p\right) \Big/ \left(\bigoplus_p \mathbb{F}_p\right)$, where p runs through all rational primes. Thus, an element of $\mathcal{A}$ is represented by a family $(a_p)_p$ of elements $a_p \in \mathbb{F}_p$, and two families $(a_p)_p$ and $(b_p)_p$ represent the same element of $\mathcal{A}$ if and only if $a_p = b_p$ for all but finitely many rational primes p.

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