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Perspectives on mock modular forms

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Abstract

Mock modular forms have played many prominent roles in number theory and other areas of mathematics over the course of the last 15 years. While the term "mock modular form" was not formally defined in the literature until 2007, we now know in hindsight that evidence of this young subject appears much earlier, and that mock modular forms are intimately related to ordinary modular and Maass forms, and Ramanujan's mock theta functions. In this expository article, we offer several different perspectives on mock modular forms – some of which are number theoretic and some of which are not – which together exhibit the strength and scope of their developing theory. They are: combinatorics, q-series and mock theta functions, mathematical physics, number theory, and Moonshine. We also describe some essential results of Bruinier and Funke, and Zwegers, both of which have made tremendous impacts on the development of the theory of mock modular forms. We hope that this article is of interest to both number theorists and enthusiasts – to any reader who is interested in or curious about the history, development, and applications of the subject of mock modular forms, as well as some amount of the mathematical details that go along with them.

1 Preamble

Modular forms have played central and far-reaching roles in number theory and mathematics over the last two centuries. Their footprints can be seen in the elliptic functions of the early 1800s, and their prominence has only risen since then. The theory and importance of their younger descendants, mock modular forms, has evolved in many analogous ways. Among the many papers written in the subject of mock modular forms are three excellent articles which are accessible to the non-specialist, and which we recommend to the readers of this article: Duke's 2014 article [54] in the *the Notices of the AMS*, Ono's 2010 article [120] in the *Notices of the AMS*, and Zagier's 2007 *Séminaire Bourbaki* article [138]. These articles highlight different aspects of the history, theory, and applications of mock modular forms, including discussions of Ramanujan's mock theta functions from 1920. The general term mock modular form was not defined in the literature until 2007 [138], and we now know in hindsight that Ramanujan's mock theta functions are among the oldest examples.

In this expository article, we begin by offering a few concrete examples of mock modular forms in Section 2, which can be viewed as companions to some familiar ordinary modular forms. We offer these examples as previews to the more formal definitions and basic properties given in Section 3. In all of the following sections, we offer several perspectives on mock modular forms from the standpoint of different areas in which mock modular forms

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