

# Accepted Manuscript

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PII: S0022-314X(16)30286-4  
DOI: <http://dx.doi.org/10.1016/j.jnt.2016.10.004>  
Reference: YJNTH 5607

To appear in: *Journal of Number Theory*

Received date: 10 April 2016  
Revised date: 23 September 2016  
Accepted date: 4 October 2016

Please cite this article in press as: C.G. Schmidt, The critical numbers of Rankin-Selberg convolutions of cohomological representations, *J. Number Theory* (2017), <http://dx.doi.org/10.1016/j.jnt.2016.10.004>

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# The Critical Numbers of Rankin-Selberg Convolutions of Cohomological Representations

Claus Günther Schmidt

November 30, 2016

## Abstract

We study the critical numbers of the Rankin-Selberg convolution of arbitrary pairs of cohomological cuspidal automorphic representations and we parametrize these critical numbers by certain 1-dimensional subrepresentations attached to the corresponding pair of finite dimensional representations of the related general linear groups.

## 1 Introduction

For arbitrary natural numbers  $n$  and  $m$  let  $\pi$  and  $\sigma$  denote cuspidal automorphic representations of  $GL_n(\mathbb{A})$  and  $GL_m(\mathbb{A})$  respectively over the adèle ring  $\mathbb{A}$  of  $\mathbb{Q}$ . Jacquet, Piatetski-Shapiro and Shalika [4] introduced for such pairs  $(\pi, \sigma)$  an  $L$ -function  $L(\pi, \sigma, s)$  which up to a few special cases is an entire function of the variable  $s$ . In analogy with Deligne's notion of *critical values* of motivic  $L$ -functions [2] we would like to study the values of  $L(\pi, \sigma, s)$  at *critical numbers*  $t \in \frac{n-m}{2} + \mathbb{Z}$  and in particular their arithmetic properties. Assuming that  $\pi$  and  $\sigma$  are cohomological in the case  $m = n - 1$  these critical values are quite well understood by [7]. See also Januszewski's contributions [5] for totally real number fields. The assumption says that there are finite-dimensional irreducible rational representations  $M_\mu$  and  $M_\nu$  of  $GL_n$  and  $GL_m$  respectively of highest weights  $\mu$  and  $\nu$  with a certain purity property such that for the infinity components  $\pi_\infty$  and  $\sigma_\infty$  the representations  $\pi_\infty \otimes M_\mu$  and  $\sigma_\infty \otimes M_\nu$  have non-trivial relative Lie algebra cohomology, i.e. we have

$$(1.1) \quad H^\bullet(\mathfrak{gl}_n, K_{n,\infty}; \pi_\infty \otimes M_{\mu,\mathbb{C}}) \neq 0$$

and

$$(1.2) \quad H^\bullet(\mathfrak{gl}_m, K_{m,\infty}; \sigma_\infty \otimes M_{\nu,\mathbb{C}}) \neq 0,$$

where for a natural number  $n$  as usual  $\mathfrak{gl}_n$  denotes the Lie algebra of  $GL_n(\mathbb{R})$ ,  $K_{n,\infty} = SO_n(\mathbb{R})Z_n^+(\mathbb{R})$  and  $Z_n^+(\mathbb{R})$  is the subgroup of matrices of positive determinant in the center  $Z_n(\mathbb{R})$  of  $GL_n(\mathbb{R})$ . For a given weight  $\mu$  the set of representations  $\pi$  with this property is usually denoted by  $\text{Coh}(\mu)$ . The treatment of the critical values for  $m = n - 1$  relied on the bijection  $t \mapsto t + \frac{1}{2}$  in this case between the set  $\text{Crit}(\pi_\infty, \sigma_\infty)$  of critical numbers and the parameter set  $\text{Emb}(\nu, \tilde{\mu})$  of integers  $s$  allowing to embed the twists  $M_{\nu-s} = \det^s \otimes M_\nu$  for  $\nu - s = (\nu_1 - s, \dots, \nu_m - s)$  into the contragredient  $M_{\tilde{\mu}}$  of  $M_\mu$  considered as

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