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# The Critical Numbers of Rankin-Selberg Convolutions of Cohomological Representations

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**Abstract**

We study the critical numbers of the Rankin-Selberg convolution of arbitrary pairs of cohomological cuspidal automorphic representations and we parametrize these critical numbers by certain 1-dimensional subrepresentations attached to the corresponding pair of finite dimensional representations of the related general linear groups.

## 1 Introduction

For arbitrary natural numbers $n$ and $m$ let $\pi$ and $\sigma$ denote cuspidal automorphic representations of $GL_n(\mathbb{A})$ and $GL_m(\mathbb{A})$ respectively over the adele ring $\mathbb{A}$ of $\mathbb{Q}$. Jacquet, Piatetski-Shapiro and Shalika [4] introduced for such pairs $(\pi, \sigma)$ an $L$-function $L(\pi, \sigma, s)$ which up to a few special cases is an entire function of the variable $s$. In analogy with Deligne's notion of *critical values* of motivic $L$-functions [2] we would like to study the values of $L(\pi, \sigma, s)$ at *critical numbers* $t \in \frac{n-m}{2} + \mathbb{Z}$ and in particular their arithmetic properties. Assuming that $\pi$ and $\sigma$ are cohomological in the case $m = n - 1$ these critical values are quite well understood by [7]. See also Januszewski's contributions [5] for totally real number fields. The assumption says that there are finite-dimensional irreducible rational representations $M_\mu$ and $M_\nu$ of $GL_n$ and $GL_m$ respectively of heighest weights $\mu$ and $\nu$ with a certain purity property such that for the infinity components $\pi_\infty$ and $\sigma_\infty$ the representations $\pi_\infty \otimes M_\mu$ and $\sigma_\infty \otimes M_\nu$ have non-trivial relative Lie algebra cohomology, i.e. we have

$$(1.1) \qquad H^\bullet(gl_n, K_{n,\infty}; \pi_\infty \otimes M_{\mu,\mathbb{C}}) \neq 0$$

and

$$(1.2) \qquad H^\bullet(gl_m, K_{m,\infty}; \sigma_\infty \otimes M_{\nu,\mathbb{C}}) \neq 0,$$

where for a natural number $n$ as usual $gl_n$ denotes the Lie algebra of $GL_n(\mathbb{R})$, $K_{n,\infty} = SO_n(\mathbb{R})Z_n^+(\mathbb{R})$ and $Z_n^+(\mathbb{R})$ is the subgroup of matrices of positive determinant in the center $Z_n(\mathbb{R})$ of $GL_n(\mathbb{R})$. For a given weight $\mu$ the set of representations $\pi$ with this property is usually denoted by $\mathrm{Coh}(\mu)$. The treatment of the critical values for $m = n - 1$ relied on the bijection $t \mapsto t + \frac{1}{2}$ in this case between the set $\mathrm{Crit}(\pi_\infty, \sigma_\infty)$ of critical numbers and the parameter set $\mathrm{Emb}(\nu, \check{\mu})$ of integers $s$ allowing to embed the twists $M_{\nu-s} = \det^s \otimes M_\nu$ for $\nu - s = (\nu_1 - s, ..., \nu_m - s)$ into the contragredient $M_{\check{\mu}}$ of $M_\mu$ considered as