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THE DISTRIBUTION OF INTEGERS IN A TOTALLY REAL CUBIC FIELD

TIANYI MAO

ABSTRACT. Hecke studies the distribution of fractional parts of quadratic irrationals with Fourier expansion of Dirichlet series. This method is generalized by Behnke and Ash-Friedberg, to study the distribution of the number of totally positive integers of given trace in a general totally real number field of any degree. When the field is cubic, we show that the asymptotic behavior of a weighted Diophantine sum is related to the structure of the unit group. The main term can be expressed in terms of Größencharacter L -functions.

1. INTRODUCTION

The study of the equidistribution of the fractional part of $m\alpha$ for α irrational and $m = 1, 2, \dots$ running over the rational integers, dates back to Weyl's work [9] in 1910. Hecke [8] studied the case when α is a fixed real quadratic irrational. His key idea is using the Fourier expansion of the Dirichlet series

$$\sum_{m \geq 1} \left(\{m\alpha\} - \frac{1}{2} \right) m^{-s}$$

to estimate the Diophantine sum

$$(1.1) \quad S_1(n) = \sum_{m=1}^n \left(\{m\alpha\} - \frac{1}{2} \right).$$

Both Behnke [4] and Ash-Friedberg [1] aim at generalizing Hecke's result to an arbitrary totally real field K of degree n . In such cases, the generalization of the fractional part of $m\alpha$ is the error term in the natural geometric estimate for the number of totally positive integers of K of a given trace. They form the Dirichlet series $\varphi(s)$ whose coefficients are these errors. More specifically, let O_K be the ring of integers of K , and let $\text{Tr}(O_K)$ be generated by $\kappa > 0$. For positive multiples a of κ , let N_a denote the number of totally positive integers with trace a . There is the natural geometric estimate r_a of N_a derived from the volume of the intersection in $O_K \otimes \mathbb{R}$ of the cone of totally positive elements with the hyperplane defined by $\text{Tr}\alpha = a$. Denote the difference between the true value and the estimate by

$$(1.2) \quad E_a = N_a - r_a.$$

If a is not a multiple of κ , we set $E_a = 0$. Then we define the Dirichlet series

$$\varphi(s) = \sum_{a > 0} \frac{E_a}{a^s}.$$

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