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Approximation of the median of the gamma distribution

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Abstract

In this paper, we provide continued fraction sequences and other sequences to approximate the median of the gamma distribution. Furthermore, we consider the approximation of the Ramanujan sequence.

1 Introduction

Let $n \geq 0$ be an integer and X_n be a random variable having the $\Gamma(n+1,1)$ distribution. The median of X_n, denoted by λ_n, is the unique solution of

$$P\{X_n \leq \lambda_n\} = \frac{1}{n!}\int_0^{\lambda_n} e^{-t}t^n dt = \frac{1}{2}.$$

In recent years, many papers have appeared providing interesting properties of λ_n. Choi [1] provided the asymptotic formula

$$\lambda_n = n + \frac{2}{3} + \frac{8}{405n} - \frac{64}{5103n^2} + \frac{2944}{492075n^3} + O\left(\frac{1}{n^4}\right), \tag{1.1}$$

and he showed that there is a connection between λ_n and the Ramanujan sequence

$$\theta_n = \frac{n!}{n^n}\left(\frac{1}{2}e^n - \sum_{k=0}^{n-1}\frac{n^k}{k!}\right). \tag{1.2}$$

Adell and Jodrá [2] proved that

$$n + \frac{2}{3} + \sum_{i=1}^{6}\frac{a_i}{n^i} < \lambda_n < n + \frac{2}{3} + \sum_{i=1}^{7}\frac{a_i}{n^i}, \tag{1.3}$$

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