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BECK-CHEVALLEY CONDITION AND GOURSAT CATEGORIES

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Abstract. We characterise regular Goursat categories through a specific stability property of regular epimorphisms with respect to pullbacks. Under the assumption of the existence of some pushouts this property can be also expressed as a restricted Beck-Chevalley condition, with respect to the fibration of points, for a special class of commutative squares. In the case of varieties of universal algebras these results give, in particular, a structural explanation of the existence of the ternary operations characterising 3-permutable varieties of universal algebras.

A variety of universal algebras is called a Mal'tsev variety [25] when any pair of congruences R and S on the same algebra 2-permute, meaning that $RS = SR$. The celebrated Mal'tsev theorem asserts that the algebraic theory of such a variety is characterised by the existence of a ternary term $p(x, y, z)$ such that the identities $p(x, y, y) = x$ and $p(x, x, y) = y$ hold [22]. The weaker 3-permutability of congruences $RSR = SRS$, which defines 3-permutable varieties, is also equivalent to the existence of two ternary operations r and s such that the identities $r(x, y, y) = x$, $r(x, x, y) = s(x, y, y)$ and $s(x, x, y) = y$ hold [17]. A nice feature of 3-permutable varieties is the fact that they are congruence modular, a condition that plays a crucial role in the development of commutator theory [12, 16].

Many interesting results have been discovered in regular Mal'tsev categories [11] and in regular Goursat categories [10], which can be seen as the categorical extensions of Mal'tsev varieties and of 3-permutable varieties, respectively. The interested reader will find many properties

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