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# Annihilator-stability and Unique Generation

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## Abstract

A ring  $R$  is said to be left uniquely generated if  $Ra = Rb$  in  $R$  implies that  $a = ub$  for some unit  $u$  in  $R$ . These rings have been of interest since Kaplansky introduced them in 1949 in his classic study of elementary divisors. Writing  $\mathfrak{l}(b) = \{r \in R \mid rb = 0\}$ , a theorem of Canfell asserts that  $R$  is left uniquely generated if and only if, whenever  $Ra + \mathfrak{l}(b) = R$  where  $a, b \in R$ , then  $a - u \in \mathfrak{l}(b)$  for some unit  $u$  in  $R$ . By analogy with the stable range 1 condition we call a ring with this property left annihilator-stable. In this paper we exploit this perspective on the left UG rings to construct new examples and derive new results. For example, writing  $J$  for the Jacobson radical, we show that a semiregular ring  $R$  is left annihilator-stable if and only if  $R/J$  is unit-regular, an analogue of Bass' theorem that semilocal rings have stable range 1.

*Key Words:* Stable range, annihilator-stable rings, uniquely generated rings, von Neumann regular rings, unit-regular rings, clean rings, triangular matrix rings, pseudo-morphic rings  
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Throughout this paper every ring  $R$  is associative with unity, all modules are unitary, and  $R$ -morphisms are written opposite to the scalars. We write  $J(R)$  for the Jacobson radical,  $U(R)$  for the unit group, and  $Z_l(R)$  and  $Z_r(R)$  for the left and right singular ideals of  $R$ . We write  $I \triangleleft R$  when  $I$  is an ideal of  $R$ . For modules we write  $N \subseteq^{ess} M$  to mean that  $N$  is essential in  $M$ . The ring of  $n \times n$  matrices over  $R$  will be denoted  $M_n(R)$ . The left and right annihilators of a set  $X$  are denoted  $\mathfrak{l}(X)$  and  $\mathfrak{r}(X)$  respectively. We denote the ring of integers by  $\mathbb{Z}$  and write  $\mathbb{Z}_n$  for the ring of integers modulo  $n$ . The term “regular ring” means von Neumann regular ring. Given a ring-theoretic condition  $\mathfrak{c}$ , a ring will be called a  $\mathfrak{c}$ -ring if it is both a left  $\mathfrak{c}$ -ring and a right  $\mathfrak{c}$ -ring, with a similar convention for elements.

## 1. Introduction

There are many important conditions on an element  $a$  in a ring  $R$  that demand the existence of a certain unit. Notable examples include:

- $a \in R$  is left **uniquely generated (UG)** if  $Ra = Rb$ ,  $b \in R$ , implies  $a = ub$  for a unit  $u$  in  $R$ ;
- $a \in R$  has left **stable range 1 (SR1)** if  $Ra + Rb = R$ ,  $b \in R$ , implies  $a - u \in Rb$  for a unit  $u$  in  $R$ ;
- $a \in R$  is **unit-regular** if  $aua = a$  for some unit  $u$  in  $R$ ;
- $a \in R$  is **clean** if  $a = e + u$  where  $e^2 = e$  and  $u$  is a unit in  $R$ .

In each case a ring  $R$  is said to have one of these conditions when **every element** of  $R$  has the condition. Here is some background about these classes of rings.

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