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Frobenius splitting for some Abelian fiber spaces

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Abstract

In this paper, we study the Frobenius split property of fibrations over curves via the relative Frobenius morphism. In particular, we can classify Frobenius split Abelian fiber spaces over curves which has no wild fibers in terms of the degree of the top higher direct image $R^{n-1}\pi_*\mathcal{O}_X$, the multiplicities of multiple fibers, non-ordinary points, and the characteristic of the base field.

Keywords: Frobenius splitting, Abelian fiber space, elliptic fibration
2010 MSC: 14J27, 14A35

1. Introduction

In this paper, we discuss Frobenius split varieties, i.e., varieties whose absolute Frobenius morphism splits. We will classify Frobenius split Abelian fiber spaces over curves which has no wild fibers in terms of the degree of the top higher direct image, the multiplicities of multiple fibers, non-ordinary points, and the characteristic of the base field.

The notion of the Frobenius split (for short, F-split) variety was introduced by Mehta and Ramanathan [MR, Definition 2] to investigate the cohomology of Schubert varieties in positive characteristic. The F-split property is a global condition on projective varieties in positive characteristic, and F-split varieties satisfy some nice properties. For example, F-split varieties satisfy the Kodaira vanishing theorem in positive characteristic.

After the introduction of F-split variety in [MR, Definition 2], F-split varieties appeared in many areas of algebraic geometry. In particular, F-split varieties are useful for studying birational geometry and representation theory in positive characteristic. Moreover, Mehta and Srinivas [MS, Lemma 1.1] showed the relationship between the notion of F-split property and the Bloch-Kato ordinarity under the assumption that the cotangent bundle is trivial.

When we have a morphism $\pi : X \to Y$ between smooth projective varieties, it is a natural question of whether the F-split property of X (resp. Y) is inherited by that of Y (resp. X). If π satisfies $\pi_*\mathcal{O}_X = \mathcal{O}_Y$ and X is F-split, then Y is also F-split; see Lemma 2.4. In general, when we have a morphism $\pi : X \to Y$, the F-split property of X often implies strong conditions not only Y but also the fibers of π. In fact, if X is F-split, then the general fibers of π are also F-split by a similar argument in [GLSTZZ, Theorem 2.1].

The simplest nontrivial families of varieties are ruled surfaces. For a ruled surface $\pi : X \to C$, X being F-split can be characterized by the following.

Fact 1.1. *[GT, Proposition 3.1] [MS1, Remark 1] Let $\pi : X \to C$ be a relatively minimal ruled surface over an algebraically closed field k of positive characteristic $p > 0$. Suppose that X is*

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