

Accepted Manuscript

Combinatorial presentation of multidimensional persistent homology

W. Chacholski, M. Scolamiero, F. Vaccarino

PII: S0022-4049(16)30147-5
DOI: <http://dx.doi.org/10.1016/j.jpaa.2016.09.001>
Reference: JPAA 5533

To appear in: *Journal of Pure and Applied Algebra*

Received date: 9 June 2012
Revised date: 22 August 2016

Please cite this article in press as: W. Chacholski et al., Combinatorial presentation of multidimensional persistent homology, *J. Pure Appl. Algebra* (2016), <http://dx.doi.org/10.1016/j.jpaa.2016.09.001>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



COMBINATORIAL PRESENTATION OF MULTIDIMENSIONAL PERSISTENT HOMOLOGY

W. CHACHOLSKI¹ M. SCOLAMIERO F. VACCARINO²

ABSTRACT. A multifiltration is a functor indexed by \mathbb{N}^r that maps any morphism to a monomorphism. The goal of this paper is to describe in an explicit and combinatorial way the natural \mathbb{N}^r -graded $R[x_1, \dots, x_r]$ -module structure on the homology of a multifiltration of simplicial complexes. To do that we study multifiltrations of sets and R -modules. We prove in particular that the \mathbb{N}^r -graded $R[x_1, \dots, x_r]$ -modules that can occur as R -spans of multifiltrations of sets are the direct sums of monomial ideals.

1. INTRODUCTION

Let \mathbb{N}^r be the poset of r -tuples of natural numbers with partial order given by $(v_1, \dots, v_r) \leq (w_1, \dots, w_r)$ if and only if $v_i \leq w_i$ for all $1 \leq i \leq r$. Given a small category \mathcal{C} , a functor $F: \mathbb{N}^r \rightarrow \mathcal{C}$ is called a **multifiltration** if, for any $v \leq w$ in \mathbb{N}^r , the map $F(v \leq w): F(v) \rightarrow F(w)$ is a monomorphism. Multifiltrations with values in the category of simplicial complexes, are the main objects we are studying in this article. By applying homology with coefficients in a ring R to a multifiltration of simplicial complexes $F: \mathbb{N}^r \rightarrow \text{Spaces}$ we obtain a functor $H_n(F, R): \mathbb{N}^r \rightarrow R\text{-Mod}$ with values in the category of R -modules. The category of functors indexed by \mathbb{N}^r with values in $R\text{-Mod}$ is equivalent to the category of \mathbb{N}^r -graded modules over the polynomial ring $R[x_1, \dots, x_r]$. One aim of this paper is to describe this $R[x_1, \dots, x_r]$ -module structure on $H_n(F, R)$ in a way that is suitable for calculations. One efficient way of doing it would be to give the minimal free presentation of $H_n(F, R)$ in terms of the multifiltration $F: \mathbb{N}^r \rightarrow \text{Spaces}$. This however we are unable to do directly. Instead we are going to describe two homomorphisms of finitely generated and free \mathbb{N}^r -graded $R[x_1, \dots, x_r]$ -modules $\mathbf{A} \rightarrow \mathbf{B} \rightarrow \mathbf{C}$ whose composition is the zero homomorphism (this sequence is a chain complex), and $H_n(F, R)$ is isomorphic to the homology of this complex. Since the modules involved are finitely generated and free and the homomorphisms preserve grading, these homomorphisms are simply given by matrices of elements in R . In our case the coefficients of the matrices are either 1, -1 or 0 and they can be explicitly expressed in terms of the multifiltration (we give a polynomial time procedure of how to do that in Section 5). One can then use standard computer algebra packages to study algebraic invariants of the module $H_n(F, R)$, in particular one can get its minimal free presentation as well as a minimal resolution, the set of Betti numbers and the Hilbert function [4, 5]. These invariants can be used then for topological

¹Partially supported by Göran Gustafsson Stiftelse and VR grant 2009-6102.

²Partially supported by the TOPDRIM project funded by the Future and Emerging Technologies program of the European Commission under Contract IST-318121.

Date: September 29, 2016.

Download English Version:

<https://daneshyari.com/en/article/5772912>

Download Persian Version:

<https://daneshyari.com/article/5772912>

[Daneshyari.com](https://daneshyari.com)