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Surjective Jordan maps and Jordan triple maps

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ABSTRACT

Surjective Jordan maps and Jordan triple maps on $B(X)$ are characterized.

1. Introduction

Throughout, all algebras and vector spaces will be over $\mathbb{F}$, where $\mathbb{F}$ is either the real field $\mathbb{R}$ or the complex field $\mathbb{C}$. Given a Banach space X with topological dual X^*, by $B(X)$ we mean the algebra of all bounded linear operators on X. Consider a map $\phi : B(X) \to B(X)$. If

$$\phi(AB + BA) = \phi(A)\phi(B) + \phi(B)\phi(A) \text{ for all } A, B \in B(X),$$

then we call ϕ a *Jordan map*; if

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$$\phi(ABA) = \phi(A)\phi(B)\phi(A) \text{ for all } A, B \in B(X),$$

then we call ϕ a *Jordan triple map*.

In [8,9], the second author showed that a bijective Jordan (triple) map of $B(X)$ is additive. Various generalizations are available ([1,2,10,4,6,5]). Among others, one direction of generalizations is to weaken the bijectivity assumption. For example, Lešnjak and Sze in [6] characterized the injective Jordan triple maps on matrix algebras and Kuzma in [5] gave a complete description of Jordan triple maps modulo those Jordan triple maps that annihilate all singular matrices.

In this paper, along this line we will characterize surjective Jordan maps and Jordan triple maps. Recall that a map $T : X \to X$ is called semilinear if it is additive and there is an automorphism $h : \mathbb{F} \to \mathbb{F}$ such that $T(\lambda x) = h(\lambda)x$ for all $x \in X$ and $\lambda \in \mathbb{F}$. Now our main result reads as follows.

Theorem 1.1. *Let X be a Banach space with dim $X > 2$. Let $\phi : B(X) \to B(X)$ be a surjective Jordan map or a surjective Jordan triple map. Then either ϕ vanishes at each operator of rank one, or there is a scalar $c = 1$ if ϕ is a Jordan map and $c \in \{1, -1\}$ if ϕ is a Jordan triple map such that one of the following holds:*

(i) *There is a semilinear bijection $T : X \to X$ such that*

$$\phi(A) = cTAT^{-1}, \ A \in B(X).$$

Moreover, if X is infinite dimensional, then T is bounded and linear or conjugate-linear.

(ii) *The space X is reflexive and there is a semilinear bijection $T : X^* \to X$ such that*

$$\phi(A) = cTA^*T^{-1}, \ A \in B(X).$$

Moreover, if X is infinite dimensional, then T is bounded and linear or conjugate-linear.

The proof will be given in the following two sections. We remark that for finite dimensional X the classification of Jordan triple maps which do not annihilate all rank-one operators has been given in [5]. Also, for Jordan triple maps, the similar results can be obtained on subalgebras of $B(X)$ which contain all finite rank operators.

We now turn to introduce some notations. For $x \in X$ and $f \in X^*$, the operator $x \otimes f$ is defined as the map $y \mapsto f(y)x$. Obviously, $x \otimes f$ is of rank one if and only if none of x and f is zero. By $\mathcal{F}_1(X)$, $\mathcal{N}_1(X)$ and $\mathcal{P}(X)$ we denote the set of all operators of rank one, the set of all nilpotent operators of rank one, and the set of all idempotent operators, respectively.