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Best unbiased estimates for parameters of three-level multivariate data with doubly exchangeable covariance structure

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ABSTRACT

The article addresses the best unbiased estimators of doubly exchangeable covariance structure, an extension of block exchangeable covariance structure, for three-level multivariate data. Under multivariate normality, the free-coordinate approach is used to obtain linear and quadratic estimates for the model parameters that are unbiased, sufficient, complete and consistent. Data from a clinical trial study is analyzed to illustrate the application of the obtained results.

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Coordinate free approach
Unstructured mean vector

1. Introduction

Multi-level multivariate observations are becoming increasingly visible across all fields of biomedical, medical and engineering among many others these days. This article deals with the estimation and best unbiased estimators of doubly exchangeable covariance structure (defined in Section 2) for three-level multivariate observations (m dimensional observation vector repeatedly measured at u locations and over v time points). Consider an example from a clinical trial of glaucoma. Glaucoma is a group of eye diseases that lead to the damage of the optic nerve. Over the years, numerous investigators have studied the characteristics of individuals who have glaucoma. Those studies identified several factors such as intraocular pressure (IOP), and central corneal thickness (CCT), useful in the diagnosis of glaucoma. Measurements of intraocular pressure (IOP) and central corneal thickness (CCT) are obtained from both the eyes (sites), each at three time points at an interval of three months for 30 patients. It is clear that for this data set $m = 2$, $u = 2$ and $v = 3$. This example will be used later in Section 5 for an illustrative purpose. Our main intention of the analysis of this data set is to illustrate the proposed method rather than giving any insight into the data set itself.

For this data set the unstructured variance–covariance matrix is (12×12)-dimensional, and therefore the number of unknown parameters in the unstructured variance–covariance matrix is 78. As a result, estimation of this unstructured variance–covariance matrix is not possible or not stable for small sample situations. Therefore, an assumption of doubly exchangeable (DE) covariance structure is necessary for small sample situation. DE covariance structure provides a substantial reduction in the number of unknown covariance parameters to just 9, and thus, may help in providing the correct information about the true association of the three-level multivariate data with small samples.

For three-level multivariate observations both u and v must be greater than 1; i.e., both $u > 1$ and $v > 1$. If either $u = 1$ or $v = 1$, the data become two-level or doubly multivariate with block exchangeable or blocked compound symmetry (BCS) covariance structure, and finally if both $u = 1$ and $v = 1$, the data just become classical multivariate data with an unstructured variance–covariance matrix. If $m = 1$ with either $u = 1$ or $v = 1$, the data also become classical multivariate data, but with a compound symmetry covariance structure.

Doubly exchangeable covariance structure was first studied by Roy and Leiva [14] in the context of classification rules for three-level multivariate data. Later these two authors wrote a series of articles on classification rules for three-level multivariate data with different covariance structures and with different mean vector structures: among them Leiva and Roy [10–12] are worth mentioning. Coelho and Roy [1] studied hypothesis testing problem of this DE covariance structure. Roy and Fonseca [16] studied this DE covariance structure while developing general linear model for three-level multivariate