

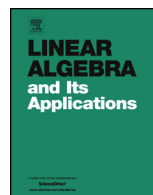


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Eigenvalue multiplicity in quartic graphs

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ABSTRACT

Let G be a connected quartic graph of order n with μ as an eigenvalue of multiplicity k . We show that if $\mu \notin \{-1, 0\}$ then $k \leq (2n-5)/3$ when $n \leq 22$, and $k \leq (3n-1)/5$ when $n \geq 23$. If $\mu \in \{-1, 0\}$ then $k \leq (2n+2)/3$, with equality if and only if $G = K_5$ (with $\mu = -1$) or $G = K_{4,4}$ (with $\mu = 0$).

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1. Introduction

Let G be a regular graph of order n with μ as an eigenvalue of multiplicity k , and let $t = n - k$. Thus the corresponding eigenspace $\mathcal{E}(\mu)$ of a $(0, 1)$ -adjacency matrix A of G has dimension k and codimension t . From [1, Theorem 3.1], we know that if $\mu \notin \{-1, 0\}$ then $k \leq n - \frac{1}{2}(-1 + \sqrt{8n+9})$, equivalently $k \leq \frac{1}{2}(t+1)(t-2)$. For connected quartic graphs, a bound which is linear in t follows easily from the equation $\text{tr}(A) = 0$. To see this, we suppose that $k \geq \frac{1}{2}n$, i.e. $k \geq t$. Then G is non-bipartite; also μ is an integer,

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for otherwise it has an algebraic conjugate which is a second eigenvalue of multiplicity k . It follows that if G is a connected quartic graph then $\mu \in \{-3, -2, 1, 2, 3\}$ (see [6, Sections 1.3 and 3.2]). Let d be the mean of the eigenvalues other than 4 and μ , so that $4 + k\mu + (n - k - 1)d = 0$. We have $-4 < d < 4$, and so:

- (a) if $\mu = -3$ then $k < \frac{4}{7}n$, i.e. $k < \frac{4}{3}t$;
- (b) if $\mu = -2$ then $k < \frac{2}{3}n$, i.e. $k < 2t$;
- (c) if $\mu = 1$ then $k < \frac{4}{5}n - \frac{8}{5}$, i.e. $k < 4t - 8$;
- (d) if $\mu = 2$ then $k < \frac{2}{3}n - \frac{4}{3}$, i.e. $k < 2t - 4$;
- (e) if $\mu = 3$ then $k < \frac{4}{7}n - \frac{8}{7}$, i.e. $k < \frac{4}{3}t - \frac{8}{3}$.

We show first that $k \leq 2t - 5$ whenever $\mu \notin \{-1, 0\}$. Then k is at most $\lfloor (2n - 5)/3 \rfloor$, a bound which is sharp for $n = 6, 9, 12$. The arguments are somewhat different from those in the paper [9], where a corresponding bound for cubic graphs was established. Section 2 contains the required results on star complements, while Section 3 provides details of the proof. It is quickly established that the bound holds when $t > 9$ or $n > 23$, and subsequently we are able to improve the bound to $(3n - 1)/5$ when $n \geq 23$. The large number of quartic graphs of order ≤ 23 justifies our case-by-case analysis when $t \leq 9$: the cases $n > 17$ are relatively easy to deal with, but there are already 86221634 connected quartic graphs of order 17 [8, Sequence A006820]. In Section 4 we show that when $\mu \in \{-1, 0\}$ we have $k \leq (2n + 2)/3$, with equality if and only if $G = K_5$ (with $\mu = -1$) or $G = K_{4,4}$ (with $\mu = 0$).

2. Preliminaries

Let G be a graph of order n with μ as an eigenvalue of multiplicity k . A *star set* for μ in G is a subset X of the vertex-set $V(G)$ such that $|X| = k$ and the induced subgraph $G - X$ does not have μ as an eigenvalue. In this situation, $G - X$ is called a *star complement* for μ in G . The fundamental properties of star sets and star complements are established in [6, Chapter 5]. We shall require the following results where we write $u \sim v$, meaning that vertices u and v are adjacent. For any $U \subseteq V(G)$, we write G_U for the subgraph of G induced by U , and $\Delta_U(v)$ for the set $\{u \in U : u \sim v\}$. For the subgraph H of G it is convenient to write $\Delta_H(v)$ for $\Delta_{V(H)}(v)$.

Theorem 2.1. (See [6, Theorem 5.1.7]) *Let X be a set of k vertices in G and suppose that G has adjacency matrix $\begin{pmatrix} A_X & B^\top \\ B & C \end{pmatrix}$, where A_X is the adjacency matrix of G_X .*

(i) *Then X is a star set for μ in G if and only if μ is not an eigenvalue of C and*

$$\mu I - A_X = B^\top (\mu I - C)^{-1} B. \tag{1}$$

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