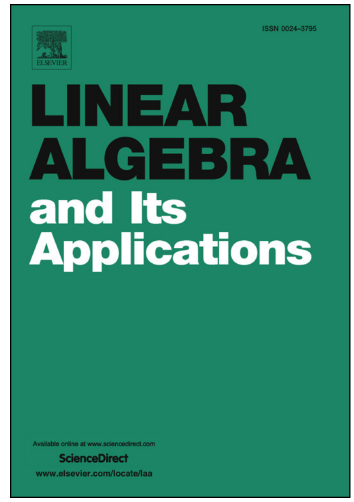

Matrix polynomials: Factorization via bisolvents

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PII: S0024-3795(17)30270-7
DOI: http://dx.doi.org/10.1016/j.laa.2017.04.033
Reference: LAA 14142

To appear in: *Linear Algebra and its Applications*

Received date: 10 December 2014
Accepted date: 24 April 2017

Please cite this article in press as: N. Cohen, E. Pereira, Matrix polynomials: Factorization via bisolvents, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.04.033

Matrix Polynomials: Factorization via Bisolvents

Nir Cohen[1], **Edgar Pereira**[1]

Abstract: *We reconsider the classification of all the factorizations of a matrix polynomial P as $P = QR$ with Q a matrix polynomial and $R(\lambda) = \lambda T - S$ a regular matrix pencil. It is shown that the entire classification problem can be reduced to the simpler classification of factors R with commuting coefficients S, T. It is then shown that, for these commuting factors, S and T must satisfy a certain algebraic equation which we call the bisolvent equation. This extends the generalized Bézout theorem which associates monic factors $\lambda I - S$ with solutions S of a solvent equation.*

In case P is regular, the classification of commuting pairs (S, T) of this type (up to left equivalence) is described in terms of enlarged standard pairs, following a well known approach. Under a non-derogatory generic condition on the roots of P, the number of such pairs associated with degree-minimal factorizations is finite, and admits explicit description in terms of Jordan chains.

MSC2010: *15A23, 15A21 , 15A22*

Keywords: *Matrix polynomial, solvent, standard pair, commuting matrices, factorization, Weierstrass form.*

1 Introduction

There is interest in extending the fundamental theorem of algebra, from polynomials over an algebraically closed field $\mathbb{F}$, to matrix-valued polynomials $P(\lambda) = \sum_{j=0}^{\pi} \lambda^j P_j$ over $\mathbb{F}$. Here, one needs to reconsider the fundamental notions of *factorability* and *roots*, due to the lack of commutativity, and to re-evaluate the equivalence between them.

As far as *factorability* is concerned, the existence and uniqueness of the factorization are no more guaranteed, irreducible factors may be nonlinear, and the sum of degrees of the factors may exceed the degree of their product. Known results are either based on spectral analysis, as summarized

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