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Some methods of construction of a common Lyapunov solution to a finite set of complex systems ☆

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In this work, two methods of determining a common Lyapunov solution for a finite number of complex matrices are proposed. The first one is an extension to the complex case of Büyükköroğlu's result dedicated to real matrices of order three whereas the second one, extending and completing a very recent paper of Gumus and Xu, can be applied to a finite set of complex matrices of arbitrary order. As special cases, some known results as well as new ones concerning the common Lyapunov solution problem for complex triangular systems are derived. Numerical examples are presented to illustrate and to compare the results.
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1. Introduction

If there exists, for a given set of matrices $\mathcal{A}$, a positive definite Hermitian matrix P such that the matrix $A^*P + PA$ is negative definite for every $A \in \mathcal{A}$, then P is said to be *a common Lyapunov solution* for $\mathcal{A}$. If P is diagonal then it is called *a common diagonal Lyapunov solution* for $\mathcal{A}$ and $\mathcal{A}$ is then said to be *simultaneously* (or *uniformly*) *diagonally stable*.

The problem of the common Lyapunov solution, motivated by the stability analysis of switched linear time-invariant systems (e.g. Liberzon [11]), is undertaken in the literature very frequently (for a brief survey of results in this field see [13] or a more recent [10]). Unfortunately, it can be effectively solved only in a very few special cases, most of which concern real matrices of small orders (see [2,4,5,16,17] and the references therein). On the other hand, there are some important issues arising in a very natural way where systems described by complex matrices occur. This makes the complex case very important both in theory and in practice.

1.1. Why complex systems?

One of the motivations for studying the existence of a common Lyapunov solution for complex matrices comes from the stability analysis of fractional systems that have recently attracted lots of attention in the control theory literature.

A typical fractional-order linear time-invariant system has the form

$$x^{(\alpha)}(t) = Ax(t), \tag{1}$$

where A is a given n-by-n matrix and $\alpha \in [1,2)$ is an order of the system. Matignon [14] proved that system (1) is asymptotically stable iff the following condition holds

$$\min_{i=1,\ldots,n} |\arg(\lambda_i(A))| > \alpha\frac{\pi}{2}, \tag{2}$$

i.e. iff all the eigenvalues of the matrix A lie in the sector shadowed in Fig. 1a). As A is real, its eigenvalues lie symmetrically relative to the real axis on the complex plane and thus condition (2) holds iff the eigenvalues of A lie in the sector shadowed in Fig. 1b). Geometrically, the eigenvalues of the matrix $(\cos\beta + i\sin\beta)A$ are those of A rotated by the angle β relative to the origin. Thus, all eigenvalues of A lie in sector shadowed in Fig. 1b) if and only if the matrix $\widetilde{A} = \left(\sin\frac{\pi}{2}\alpha + i\cos\frac{\pi}{2}\alpha\right)A$ has all its eigenvalues in the open left-half of the complex plane. This allows us to conclude that the stability of fractional system (1) is equivalent to the stability of the first-order system $y'(t) = \widetilde{A}y(t)$. Note that if A is real then $\widetilde{A}$ has complex entries.

Suppose now that for some reasons (that will not be discussed here but are well known to those who are familiar with the robust stability theory) the matrix A describing

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