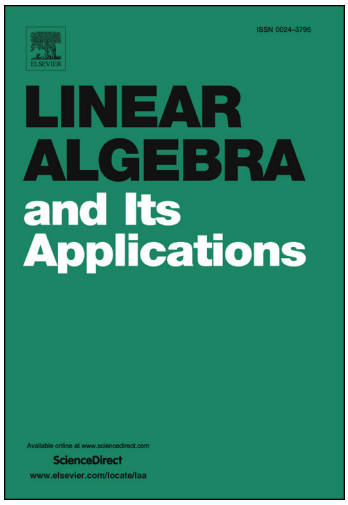

Hurwitz matrices of doubly infinite series

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Hurwitz matrices of doubly infinite series[☆]

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Abstract

This paper aims at extending the criterion that the quasi-stability of a polynomial is equivalent to the total nonnegativity of its infinite Hurwitz matrix. We give a complete description of functions generating doubly infinite series with totally nonnegative Hurwitz and Hurwitz-type matrices (in a Hurwitz-type matrix odd and even rows come from two distinct power series). The corresponding result for singly infinite series is known: it is based on a certain factorization of Hurwitz-type matrices, which is unavailable in the doubly infinite case. A necessary condition for total nonnegativity of generalized Hurwitz matrices follows as an application.

Keywords: Total positivity, Pólya frequency sequence, Hurwitz matrix, Generalized Hurwitz matrix, Doubly infinite series
2010 MSC: 30C15, 30B10, 40A05

1. Introduction

Definition. A doubly (*i.e.* two-way) infinite sequence $\left(f_n\right)_{n=-\infty}^{\infty}$ is called *totally positive* if all minors of the (four-way infinite) Toeplitz matrix

$$\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \cdots & f_0 & f_1 & f_2 & f_3 & f_4 & \cdots \\ \cdots & f_{-1} & f_0 & f_1 & f_2 & f_3 & \cdots \\ \cdots & f_{-2} & f_{-1} & f_0 & f_1 & f_2 & \cdots \\ \cdots & f_{-3} & f_{-2} & f_{-1} & f_0 & f_1 & \cdots \\ \cdots & f_{-4} & f_{-3} & f_{-2} & f_{-1} & f_0 & \cdots \\ \iddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} =: T(f), \quad \text{where} \quad f(z) := \sum_{n=-\infty}^{\infty} f_n z^n$$

are nonnegative (*i.e.* the matrix is *totally nonnegative*).

Note that the indexation of four-way infinite matrices affects the multiplication. Here we adopt the following convention: the uppermost row and the leftmost column, which appear in representations of such matrices, have the index 1 unless another is stated explicitly.

The total nonnegativity of the corresponding Toeplitz matrices is a characteristic property of power series converging to functions of a very specific form:

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