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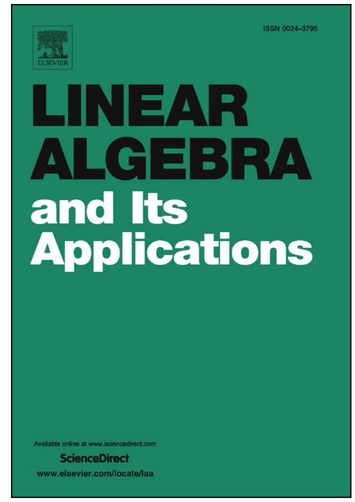

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On the normalized spectrum of threshold graphs

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Abstract

In this article we investigate normalized adjacency eigenvalues (simply normalized eigenvalues) and normalized adjacency energy of connected threshold graphs. A threshold graph can always be represented as a unique binary string. Certain eigenvalues are obtained directly from its binary representation and the rest of the eigenvalues are evaluated from its normalized equitable partition matrix. Finally, we characterize threshold graphs with at most five distinct eigenvalues.

AMS classification 05C50.
Keywords normalized eigenvalues; pineapple graph; Randić matrix; threshold graphs; normalized Laplacian

1 Introduction

In this paper, we only consider simple, connected, undirected finite graphs. Let $\Gamma = (V, E)$ be an n vertex graph with $V = \{1, 2, \ldots, n\}$. Two vertices $i, j \in V$ are called neighbors, $i \sim j$, when they are connected by an edge in E. Let d_i denote the degree of a vertex i. Let A be the adjacency matrix [8] of Γ and let D be the diagonal matrix of vertex degrees of Γ. The *normalized adjacency* matrix $\mathcal{A}$ of Γ is defined by $\mathcal{A} = D^{-1}A$ which is similar to the matrix $R = D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$, called the *Randić matrix* [4] of Γ. Thus the matrices R and $\mathcal{A}$ have same eigenvalues. The matrix $\mathcal{A}$ is a row-stochastic matrix, often called the *transition matrix* of Γ. For any function $f : V(\Gamma) \longrightarrow \mathbb{R}$, $\mathcal{A}$ is given by

$$\mathcal{A}f(i) = \frac{1}{d_i} \sum_{j \sim i} f(j), \text{ for all } i \in V(\Gamma).$$

Furthermore, $\mathcal{A}$ is self-adjoint with respect to the inner product defined by

$$\langle u, v \rangle = \sum_i d_i u(i) v(i).$$