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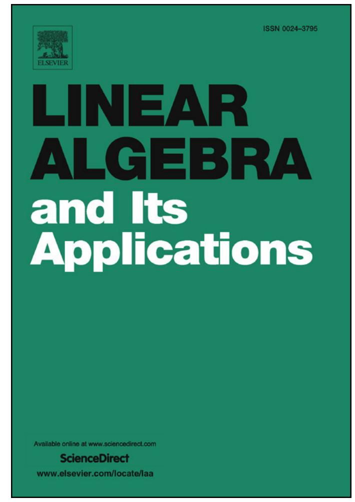

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Combinatorics related to Higman's conjecture I: Parallelogramic digraphs and dispositions

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Abstract

In this paper we introduce a kind of directed graphs (digraphs) arranged in shifted rows of different lengths, which arise in a natural way related to problems of finding the number of certain families of canonical primitive connected cellular matrices of the p-Sylow $\mathfrak{G}_n$ of $GL_n(q)$ formed by the upper unitriangular matrices over the finite field with q elements. Higman's conjecture states that the number of conjugacy classes of $\mathfrak{G}_n$ is a polynomial in q. We associate a number, which we call the counter, to each directed graph, which gives additional information about the polynomial structure of the number of conjugacy classes. We focus in a family of digraphs, which we call parallelogramic digraphs, in which we have n rows of length k each one shifted one place to the right with respect to the previous one. We give explicit formulas for their counters for n up to 5. We prove also that the counters satisfy recurrence equations for fixed k when we vary n, proving thus a fact that was empirically observed by R.H. Harding and A.P. Heinz and proved by P. Sun for k up to 5. When $n > 1$, this number multiplied by $(q-1)^{nk-1}$ corresponds to the cardinality of the family of canonical cellular $nk \times nk$ matrices over the field $\mathbb{F}_q$ with n pivot lines of length k and exactly $k-1$ links connecting the pilots of the lines. We indicate other kinds of digraphs related to Higman's conjecture that establish lines of future research on this topic.

Keywords: Group theory, Combinatorics, Higman's conjecture
2000 MSC: 05E15

1. Introduction

The interplay between the algebraic properties of the representations and characters of the symmetric and classical groups and the combinatorial properties of counting bidimensional arrangments of numbers preserving inequalities from left to right and from up to down has been present for a long time. It began with the application by Frobenius of Young's tableaux to the study of the characters of symmetric groups ([3]). Young tableaux were also applied to the study of representations of some classical groups, as for instance, of $U(k), SU(k), SO(2k+1), SP(2k)$ and $SO(2k)$ in [8], or, more generally, in [12].

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