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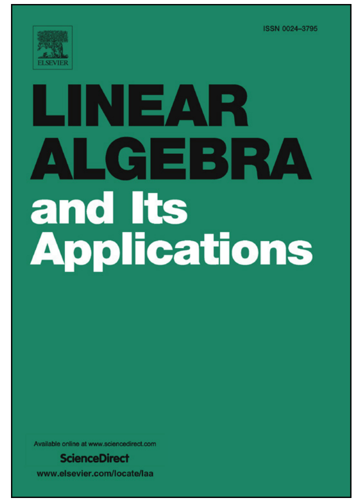

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Graphs whose distance matrix has at most three negative eigenvalues

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Abstract: Let $D(G)$ be the distance matrix of a connected simple graph G. The negative inertia of $D(G)$, denoted by $n_D(G)$, is the number of negative eigenvalues of $D(G)$. In this paper, we determine all connected graphs G whose distance matrix $D(G)$ has at most three negative eigenvalues.

Keywords: Distance matrix; D-eigenvalue; Negative inertia

AMS classification: 05C50; 15A18

1 Introduction

Throughout, only connected, undirected and simple graphs are considered. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The order of G is the number of vertices of G, which is $|V(G)|$. For two vertices u and v, the distance between u and v, denoted by $dist(u, v)$, is defined as the length of a shortest path joining u and v. The diameter of G, written as $d(G)$, is the maximum distance among all vertices of $V(G)$. The length of a smallest cycle of G, denoted by $g(G)$, is referred to as the girth of G. The set of vertices of G adjacent to a vertex u is written as $N_G(u)$. By $u \sim v$ (resp., $u \nsim v$) we mean u is adjacent (resp., not adjacent) to v. A subgraph H of G is called an induced subgraph, if H is obtained by deleting some vertices and the edges incident to them from G. As usual, we use K_n, P_n, $S_{1,n-1}$ and C_n to denote the complete graph, the path, the star and the cycle of order n, respectively. The identity matrix of order n is denoted by I_n. If the vertices of G are labelled as $V(G) = \{v_1, v_2, \dots, v_n\}$, then the distance matrix $D(G)$ of G is defined as a symmetric real matrix with $dist(v_i, v_j)$ as the (i, j)-entry. The eigenvalues of $D(G)$ are called the distance eigenvalues of G, or simply, D-eigenvalues of G. For graphs G_1 and G_2, we say G_1 is isomorphic to G_2, if G_2 can be obtained from G_1 by relabeling the vertices of G_1.

The number of negative D-eigenvalues is called the negative inertia of $D(G)$, denoted by $n_D(G)$. Generally, the inertia of $D(G)$ is defined as the triple of integers $(p_D(G), \eta_D(G), n_D(G))$, where $p_D(G)$ and $\eta_D(G)$ denote the number of positive and zero eigenvalues of $D(G)$, respectively. In [9], Graham and Pollack (1971) proved $det(D(T)) = (-1)^{n-1}(n-1)2^{n-2}$ for a tree T of order n, which implies that there is exactly one positive D-eigenvalue for a tree T, *i.e.*, $p_D(T) = 1$. From then on, many papers focused on the distance matrices of graphs, and some

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