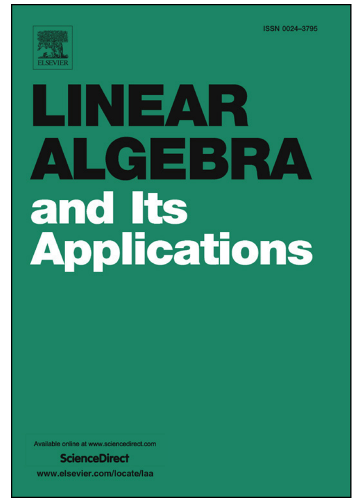
Accepted Manuscript



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PII: S0024-3795(17)30344-0
DOI: http://dx.doi.org/10.1016/j.laa.2017.05.044
Reference: LAA 14192

To appear in: *Linear Algebra and its Applications*

Received date: 25 March 2017
Accepted date: 28 May 2017

On graphs with distance Laplacian spectral radius of multiplicity $n-3$ *

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Abstract Let $\partial_1^L \geq \partial_2^L \geq \cdots \geq \partial_n^L$ be the distance Laplacian eigenvalues of a connected graph G and $m(\partial_i^L)$ the multiplicity of ∂_i^L. It is well known that the graphs with $m(\partial_1^L) = n-1$ are complete graphs. Recently, the graphs with $m(\partial_1^L) = n-2$ have been characterized by Celso et al. In this paper, we completely determine the graphs with $m(\partial_1^L) = n-3$.

Keywords: Distance Laplacian matrix; Laplacian matrix; Largest eigenvalue; Characterised by distance Laplacian spectrum

AMS subject classifications: 05C50; 05C12; 15A18

1 Introduction

In this paper we only consider simple connected graphs. Let $G = (V, E)$ be a connected graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$. The *distance* between v_i and v_j, denoted by $d_G(v_i, v_j)$, is defined as the length of a shortest path between them. The *diameter* of G, denoted by $d(G)$, is the maximum distance between any two vertices of G. The *distance matrix* of G, denoted by $\mathcal{D}(G)$, is the $n \times n$ matrix whose (i, j)-entry is equal to $d_G(v_i, v_j)$, $i, j = 1, 2, \ldots, n$. The *transmission* $Tr(v_i)$ of a vertex v_i is defined as the sum of the distances between v_i and all other vertices in G, that is, $Tr(v_i) = \sum_{j=1}^{n} d_G(v_i, v_j)$. For more details about the distance matrix we refer the readers to [1]. Aouchiche and Hansen [2] introduced the Laplacian for the distance matrix of G as $\mathcal{D}^L(G) = Tr(G) - \mathcal{D}(G)$, where $Tr(G) = diag(Tr(v_1), Tr(v_2), \ldots, Tr(v_n))$ is the diagonal matrix of the vertex transmissions in G. The eigenvalues of $\mathcal{D}^L(G)$, listed by $\partial_1^L \geq \partial_2^L \geq \cdots \geq \partial_n^L = 0$, are called the *distance Laplacian eigenvalues* of G. The multiplicity of ∂_i^L is denoted by $m(\partial_i^L)$. The distance eigenvalues together with their multiplicities is called the *distance Laplacian spectrum* of G, denoted by $Spec_{\mathcal{L}}(G)$.

The distance Laplacian matrix aroused many active studies, such as [1, 6, 10, 11]. Graphs with few distinct eigenvalues form an interesting class of graphs and possess nice combinatorial properties. With respect to distance Laplacian eigenvalues, we focus on the graphs with $m(\partial_1^L)$ being large. Denote by $\mathcal{G}(n)$ the set of connected graphs of order

*Supported by the National Natural Science Foundation of China (Grant Nos. 11671344, 11531011).

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