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Extension of diagonal stability and stabilization for continuous-time fractional positive linear systems

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A B S T R A C T

This paper considers an extension of diagonal stability for continuous fractional positive linear systems (FPLS) with the fractional order $0 < \alpha < 1$. Based on diagonal stability of *Metzler* matrix, an extension that involves the combination of a diagonal positive definite matrix and a skew-symmetric anti-diagonal matrix related a *Hurwitz* and *Metzler* matrix is established. Combining this extension and the linear matrix inequalities (LMIs) criteria of stability for fractional order systems (FOS), a necessary and sufficient condition for extension diagonal stability of FPLS is presented. A state feedback controller is given, which ensures the stabilization and positivity of the closed-loop systems. Numerical examples are provided to demonstrate the effectiveness and applicability of the proposed methods.

1. Introduction

In recent years, fractional order systems (FOS) have attracted much increasing attention. Researchers have noticed that the description of some phenomena is more accurate

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while the fractional derivatives and integrals (see [1], an introduction to the theory) are introduced. In particular, it shows that many physical phenomena having memory and genetic characteristics [2], mass diffusion and heat conduction [3], biomedical systems [4,5] can be described by modeling as fractional order systems more adequately than traditional integer models. It is worth mentioning that fractional order derivatives and integrals are powerful instruments for modeling dynamical processes, such as fractal medium [6]. For commensurate FOS, the stability and stabilization are fundamental. The powerful criteria of stability properties for fractional differential systems are proposed in [7] (classical Matignon's stability theorem), which determines the stability for FOS through the location of the state matrix eigenvalues in the complex plane. Based on Matignon's stability theorem, fractional Lyapunov direct method and the definition of Mittag–Leffler stability are discussed in [8]. Although there are huge LMI stability conditions for FOS e.g. in [9,10], major results have drawbacks that are not effective LMI conditions. Recently, an effective new LMI condition which coincides with that of integer order systems is presented in [11], based on this LMI condition, we can extend many results of integer order systems to fractional order systems. Our purpose in this paper is to extend some properties of integer positive linear systems (IPLS) to FPLS by using the new LMI condition.

Additionally, in practice, the states of a class of systems, called positive systems, are often constrained to be nonnegative. Such systems can be found in some fields: industrial engineering [12], communication networks [13], social sciences, and economics. The monographs [14,15] give an overview of properties of positive systems, one of the most important property is that, for IPLS $\dot{x} = Ax$, this system is stable iff matrix A is *Hurwitz Metzler* matrix. Another important property is the diagonal stability of IPLS, i.e., suppose that matrix A is *Metzler* and *Hurwitz*, there exist a positive definite diagonal matrix D such that $A^T D + DA$ is negative definite. And there are lots of studies about the diagonal stability of *Hurwitz Metzler* matrix [16,17]. A method that determines a given *Metzler* matrix is *Hurwitz* or not is presented in [18]. An alternative proof of the BBP result on diagonal stability is given in [19]. The existence of a diagonal common quadratic Lyapunov function for a pair of IPLS is presented in [20]. These results are all given on the basis of quadratic Lyapunov function, and for the FOS, there is no real sense of Lyapunov function. So we cannot use the general method to extent the results of the integer order to the fractional order. Nevertheless, [21] recently proves a new result on diagonal stability for *Hurwitz Metzler* matrix by duality-based arguments. This duality method also can be effectively applied to LMI condition of stability for FOS.

With the above motivations, this paper studies the diagonal stability for FPLS. The contribution of this paper is summarized as follows. Firstly, we propose an extension of diagonal stability for *Hurwitz Metzler* matrix by using the duality method. Based on this extension, sufficient and necessary conditions of stability for FPLS with diagonal solutions, are also proved. The diagonal solutions of the LMIs for FPLS involve a diagonal positive definite matrix and a skew-symmetric anti-diagonal matrix. Although the stability of the fractional order system can be simply checked by Lemmas 7 and 8