Linear Algebra and its Applications 530 (2017) 541–557

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Corrigendum

Corrigendum to "On eigenvalues of Laplacian matrix for a class of directed signed graphs" [Linear Algebra Appl. 523 (2017) 281–306] ☆

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A R T I C L E I N F O

Article history:
Received 12 May 2017
Accepted 14 June 2017
Available online 8 July 2017
Submitted by R. Brualdi

MSC:
05C50
15A18
05C20
05C22
47N70
05C82

Keywords:
Directed signed graph
Eigenvalues of Laplacian matrix

A B S T R A C T

This note corrects an error in the results of Subsection 3.1 in authors' paper "On Eigenvalues of Laplacian Matrix for a Class of Directed Signed Graphs", which appeared in Linear Algebra and its Applications 523 (2017), 281–306.

DOI of original article: http://dx.doi.org/10.1016/j.laa.2017.02.029.

☆ The work of I. Shames and D. Nešić was supported by Discovery Project under Grant DP170104099.

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http://dx.doi.org/10.1016/j.laa.2017.06.023

1. Preliminaries and graph notation

Throughout this paper, $I_N \in \mathbb{R}^{N\times N}$, $\mathbf{1}_N \in \mathbb{R}^N$, and $\mathbf{0}_N \in \mathbb{R}^N$ denote the $N \times N$ identity matrix, the N-dimensional vectors containing 1, and 0 in every entry, respectively. The standard bases in $\mathbb{R}^N$ are represented by $\{\mathbf{e}_1, \dots, \mathbf{e}_N\}$ where $\mathbf{e}_i$ is the ith column of I_N. The 2-norm of a vector $x \in \mathbb{R}^N$ is shown by $\|x\|$. For a complex variable, vector or matrix, $\Re(\cdot)$ and $\Im(\cdot)$ stand for the real and imaginary parts. For a matrix $A \in \mathbb{R}^{N\times N}$, $\text{Spec}(A) = \{\lambda_i(A)\}_{i=1}^N$ denotes the set of eigenvalues of A where $\Re(\lambda_1) \le \Re(\lambda_2) \le \cdots \le \Re(\lambda_N)$. An eigenvalue $\lambda_i(A)$ is called semisimple if its algebraic and geometric multiplicities are equal. The operator diag$(\cdot)$ constructs a block diagonal matrix from its arguments. The entry in the ith row and jth column of a matrix A is represented by $[A]_{ij}$, while the ith entry of a vector x is denoted by $[x]_i$. For a set $\mathcal{A}$, its cardinality is denoted by $|\mathcal{A}|$.

A weighted directed signed graph $\mathcal{G}$ is represented by the triple $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$ where $\mathcal{V} = \{1, \dots, N\}$ is the nodes set, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{W} : \mathcal{V} \times \mathcal{V} \to \mathbb{R}$ is a weight function that maps each $(i, j) \in \mathcal{E}$ to a nonzero scalar a_{ij} and returns 0 for all other $(i, j) \notin \mathcal{E}$. The adjacency matrix $A \in \mathbb{R}^{N\times N}$ captures the interconnection between the nodes in the graph where $[A]_{ij} = a_{ij} \neq 0$ iff $(i, j) \in \mathcal{E}$. For the edge (i, j), we follow the definition corresponding to a sensing convention which indicates that node i receives information form node j or equivalently, the node j influences the node i; see [13] for more information. For each node $i \in \mathcal{V}$, $\mathcal{N}_i$ denotes the set of its neighbors, i.e., $\mathcal{N}(i) = \{j \mid a_{ij} \neq 0\}$.

For a given graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$ and a set $\overline{\mathcal{V}} \subseteq \mathcal{V}$, the corresponding induced subgraph is denoted by $\mathcal{G}(\overline{\mathcal{V}}, \overline{\mathcal{E}}, \overline{\mathcal{W}})$, where the set $\overline{\mathcal{E}}$ is defined as $\overline{\mathcal{E}} = \{(i, j) \in \mathcal{E} \mid i, j \in \overline{\mathcal{V}}\}$, and $\overline{\mathcal{W}} : \overline{\mathcal{V}} \times \overline{\mathcal{V}} \to \mathbb{R}$ is defined as $\overline{\mathcal{W}}(i, j) = \mathcal{W}(i, j)$. In order to categorize edges in terms of the sign of their values, we define the sets $\mathcal{E}^+ = \{(i, j) \mid a_{ij} > 0\}$, and $\mathcal{E}^- = \mathcal{E} \backslash \mathcal{E}^+ = \{(i, j) \mid a_{ij} < 0\}$. We call the edges in $\mathcal{E}^+$ and $\mathcal{E}^-$ positive edges and negative edges, respectively. Subsequently, for a signed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$, we denote the subgraph with non-negative weights by $\mathcal{G}(\mathcal{V}, \mathcal{E}^+, \mathcal{W}^+)$ where $\mathcal{W}^+ : \mathcal{V} \times \mathcal{V} \to \mathbb{R}_{\ge 0}$ is defined as $\mathcal{W}^+(i, j) = \mathcal{W}(i, j)$ for all $(i, j) \in \mathcal{E}^+$ and $\mathcal{W}^+(i, j) = 0$ for all $(i, j) \notin \mathcal{E}^+$. Similarly, for a signed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$, we denote the subgraph with non-positive weights by $\mathcal{G}(\mathcal{V}, \mathcal{E}^-, \mathcal{W}^-)$. The superposition of two signed directed graphs $\mathcal{G}_1(\mathcal{V}, \mathcal{E}_1, \mathcal{W}_1) \oplus \mathcal{G}_2(\mathcal{V}, \mathcal{E}_2, \mathcal{W}_2)$ is a new graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$ where $\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2$ and, $\mathcal{W}(i, j) = \mathcal{W}_1(i, j) + \mathcal{W}_2(i, j)$ for every $(i, j) \in \{\mathcal{V} \times \mathcal{V}\}$.

For a directed graph, the in-degree and out-degree of node i are defined as $d_i^{in} = \sum_j a_{ji}$ and $d_i^{out} = \sum_j a_{ij}$ respectively. The Laplacian matrix[1] $L \in \mathbb{R}^{N\times N}$ is defined by $L = D - A$ where $D = \text{diag}\{d_1^{out}, \dots, d_N^{out}\}$. Since the rows of the Laplacian matrix add to zero, $\mathbf{1}_N$ is

[1] The current definition of Laplacian matrix has been inspired from a large variety of applications in control such as consensus [15], security analysis of complex networks [12,11], and synchronization in networks of oscillators [10,2]. However, there is another way to define the Laplacian matrix for weighted signed graphs in which the in-degree and out-degree of node i are defined as $d_i^{in} = \sum_j |a_{ji}|$ and $d_i^{out} = \sum_j |a_{ij}|$ respectively [5].