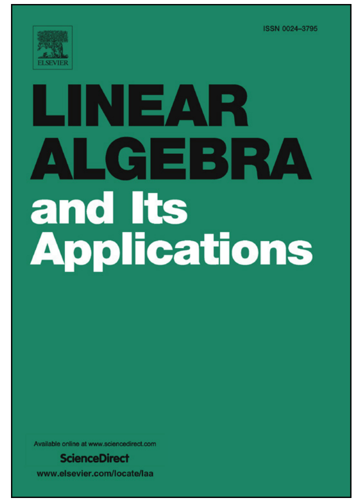
Accepted Manuscript



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PII: S0024-3795(17)30414-7
DOI: http://dx.doi.org/10.1016/j.laa.2017.07.004
Reference: LAA 14250

To appear in: *Linear Algebra and its Applications*

Received date: 15 June 2016
Accepted date: 5 July 2017

Please cite this article in press as: C. Boutsidis et al., A randomized algorithm for approximating the log determinant of a symmetric positive definite matrix, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.07.004

A Randomized Algorithm for Approximating the Log Determinant of a Symmetric Positive Definite Matrix

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Abstract

We introduce a novel algorithm for approximating the logarithm of the determinant of a symmetric positive definite (SPD) matrix. The algorithm is randomized and approximates the traces of a small number of matrix powers of a specially constructed matrix, using the method of Avron and Toledo [AT11]. From a theoretical perspective, we present additive and relative error bounds for our algorithm. Our additive error bound works for any SPD matrix, whereas our relative error bound works for SPD matrices whose eigenvalues lie in the interval $(\theta_1, 1)$, with $0 < \theta_1 < 1$; the latter setting was proposed in [HMS15]. From an empirical perspective, we demonstrate that a C++ implementation of our algorithm can approximate the logarithm of the determinant of large matrices very accurately in a matter of seconds.

1 Introduction

Given a matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, the determinant of $\mathbf{A}$, denoted by $\det(\mathbf{A})$, is one of the most important quantities associated with $\mathbf{A}$. Since its invention by Cardano and Leibniz in the late 16th century, the determinant has been a fundamental mathematical concept with countless applications in numerical linear algebra and scientific computing. The advent of Big Data, which are often represented by matrices, increased the applicability of algorithms that compute, exactly or approximately, matrix determinants; see, for example, [LZL05, ZLLW08, ZL07, dBEG08, HSD$^+$13] for machine learning applications (e.g., gaussian process regression) and [LP01, KL13, FHT08, PB97, PBGS00] for several data mining applications (e.g., spatial-temporal time series analysis).

Formal definitions of the determinant include the well-known formulas derived by Leibniz and Laplace; however, neither the Laplace nor the Leibniz formula can be used to design an efficient, polynomial-time, algorithm to compute the determinant of $\mathbf{A}$. To achieve this goal, one should rely on other properties of the determinant. For example, a standard approach would be to leverage the so-called LU matrix decomposition or the Cholesky decomposition for symmetric positive definite matrices (SPD) to get an $O(n^3)$ deterministic algorithm to compute the determinant of $\mathbf{A}$. (Recall that an SPD matrix is a symmetric matrix with strictly positive eigenvalues.)

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