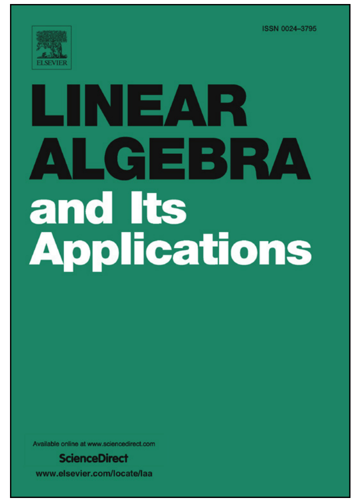

Isomorphism classes of four dimensional nilpotent associative algebras over a field

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Isomorphism classes of four dimensional nilpotent associative algebras over a field

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Abstract

In this paper we classify the isomorphism classes of four dimensional nilpotent associative algebras over a field $\mathbb{F}$, studying regular subgroups of the affine group $\mathrm{AGL}_4(\mathbb{F})$. In particular we provide explicit representatives for such classes when $\mathbb{F}$ is a finite field, the real field $\mathbb{R}$ or an algebraically closed field.

Keywords: Regular subgroup, congruent matrices, nilpotent algebra, finite field
2010 MSC: 16Z05, 16N40, 15A21, 20B35

1. Introduction

The aim of this paper is the classification of the isomorphism classes of four dimensional nilpotent associative algebras over a field $\mathbb{F}$, exploiting the properties of regular subgroups of the affine group $\mathrm{AGL}_4(\mathbb{F})$.

It is worth recalling that the affine group $\mathrm{AGL}_n(\mathbb{F})$ can be identified with the subgroup of $\mathrm{GL}_{n+1}(\mathbb{F})$ consisting of the invertible matrices having $(1,0,\ldots,0)^{\mathrm{T}}$ as first column. It follows that $\mathrm{AGL}_n(\mathbb{F})$ acts on the right on the set $\mathcal{A} = \{(1,v) : v \in \mathbb{F}^n\}$ of affine points. A subgroup R of $\mathrm{AGL}_n(\mathbb{F})$ is called regular if it acts regularly on $\mathcal{A}$, namely if for every $v \in \mathbb{F}^n$ there exists a unique element in R having $(1,v)$ as first row. Writing the elements r of a regular subgroup $R \leq \mathrm{AGL}_n(\mathbb{F})$ as

$$r = \begin{pmatrix} 1 & v \\ 0 & I_n + \delta_R(v) \end{pmatrix} = \mu_R(v), \qquad (1.1)$$

we give rise to the functions $\delta_R : \mathbb{F}^n \to \mathrm{Mat}_n(\mathbb{F})$ and $\mu_R : \mathbb{F}^n \to R$. For instance, the translation subgroup $\mathcal{T}_n$ of $\mathrm{AGL}_n(\mathbb{F})$ is a regular subgroup, corresponding to the choice $\delta_{\mathcal{T}_n}$ equal to the zero function.

We focus our attention on the set $\Delta_n(\mathbb{F})$ of the regular subgroups R of $\mathrm{AGL}_n(\mathbb{F})$ with the property that δ_R is a linear function. Notice that if R is abelian, then $R \in \Delta_n(\mathbb{F})$ (see [2, 12]); however, for $n \geq 3$, the set $\Delta_n(\mathbb{F})$

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