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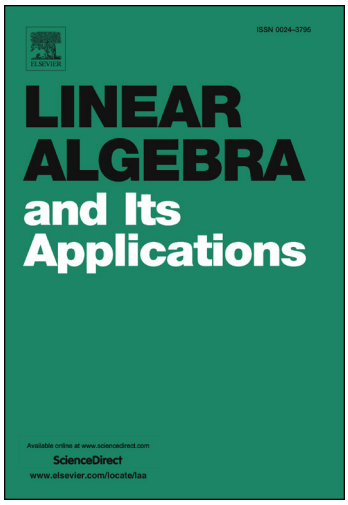

Hermitian matrices: Spectral coupling, plane geometry/trigonometry and optimisation

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Hermitian matrices : Spectral coupling, plane geometry/trigonometry and optimisation

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Abstract

The paper presents the information processing that can be performed by a general hermitian matrix when two of its distinct eigenvalues are coupled, such as $\lambda < \lambda'$, instead of considering only one eigenvalue as traditional spectral theory does. Setting $a = \frac{\lambda+\lambda'}{2} \neq 0$ and $e = \frac{\lambda'-\lambda}{2} > 0$, the information is delivered in geometric form, both metric and trigonometric, associated with various right-angled triangles exhibiting optimality properties quantified as ratios or product of $|a|$ and e. The potential optimisation has a triple nature which offers two possibilities: in the case $\lambda\lambda' > 0$ they are characterised by $\frac{e}{|a|}$ and $|a|e$ and in the case $\lambda\lambda' < 0$ by $\frac{|a|}{e}$ and $|a|e$. This nature is revealed by a key generalisation to indefinite matrices over $\mathbb{R}$ or $\mathbb{C}$ of Gustafson's operator trigonometry.

Keywords:
Spectral coupling, indefinite symmetric or hermitian matrix, spectral plane, invariant plane, catchvector, antieigenvector, midvector, local optimisation, Euler equation, balance equation, torus in 3D, angle between complex lines
2010 MSC: 15A18,
2010 MSC: 15A42,
2010 MSC: 15B57,
2010 MSC: 35Q31,
2010 MSC: 51M99

1. Spectral coupling

1.1. Introduction

In the work we present below, we focus our attention on the coupling of any two distinct real eigenvalues $\lambda < \lambda'$ of a general hermitian or sym-