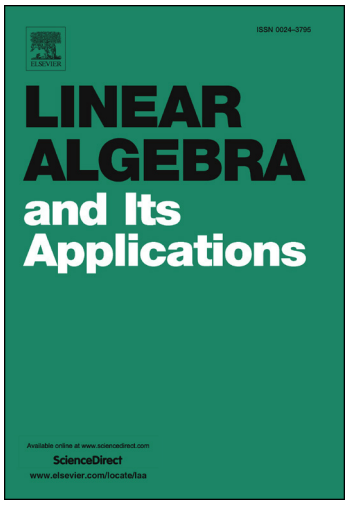

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UNIFYING A RESULT OF THOMPSON AND A RESULT OF FIEDLER AND MARKHAM ON BLOCK POSITIVE DEFINITE MATRICES

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Abstract. We prove a determinant inequality for block positive definite matrices which unifies a result of Thompson (1961) and a result of Fiedler and Markham (1994).

1. Introduction

If $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$ is a (Hermitian) positive definite matrix with diagonal blocks square, then the classical Fischer's inequality [5, p. 506] says that

$$\det H_{11} \det H_{22} \geq \det H.$$

Everitt [2] observed that if further the off-diagonal blocks are square, then a stronger inequality holds

$$\det H_{11} \det H_{22} - |\det H_{12}|^2 \geq \det H. \tag{1.1}$$

Note that the left side of (1.1) is equal to the determinant of the 2×2 compressed matrix $\begin{bmatrix} \det H_{11} & \det H_{12} \\ \det H_{21} & \det H_{22} \end{bmatrix}$.

Thompson [13] extended Everitt's result to higher number of blocks.

Theorem 1.1. *If $H = [H_{ij}]$ is positive definite with each block H_{ij} square, then*

$$\det([\det H_{ij}]) \geq \det H. \tag{1.2}$$

Over the years, various extensions and generalizations of (1.2) have been obtained in the literature, e.g., [10, 11, 14]. Analogous results have also been considered for M-matrices [6] and totally positive matrices [3].

On the other hand, Fiedler and Markham [4] revisited a theorem of Everitt, Thompson and de Pillis and proved the following inequality.

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