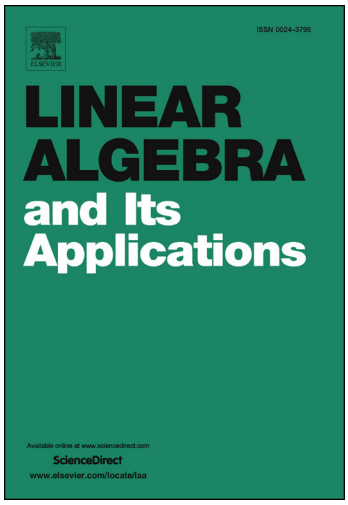

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Algebraic condition for the singularity of certain Toeplitz pencils

Wiland Schmale*

Abstract

An algebraic condition for the singularity of certain Toeplitz matrix pencils is derived which involves only the principal minors of the constant parts of the pencils. This leads to an algebraic conjecture which is equivalent to the so-called Toeplitz pencil conjecture.

keywords: Toeplitz pencil, conjecture, singularity MSC2010: 15B05, 93C05

1 Introduction

Let $T(x) = M_0 + xM_1$, where the $n \times n$-matrices M_0, M_1 are given as

$$M_0 = \begin{bmatrix} c_2 & c_1 & 0 & \dots & 0 \\ c_3 & c_2 & c_1 & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ c_n & c_{n-1} & \dots & \dots & c_1 \\ c_{n+1} & c_n & \dots & \dots & c_2 \end{bmatrix}, \; M_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \dots & \dots & 1 \\ 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 \end{bmatrix}, n \geq 2.$$

The complex numbers $c_1, c_2, \dots, c_{n+1}$ are all supposed to be **non-zero**. x is an indeterminate over $\mathbb{C}$. M_1 is the 2×2 zero-matrix if $n = 2$.

Toeplitz Pencil Conjecture (ToePC)[(1)] **([1]).** *If* $\det T(x) = 0$ *as a polynomial in* $\mathbb{C}[x]$*, i.e. all coefficients vanish, then the first two columns of* M_0 *(or* $T(x)$*) are linearly dependent. I.e. for some non zero complex number* λ *one has* $c_{k+1} = \lambda c_k$ *or equivalently* $c_{k+1} = \lambda^k c_1$ *for* $k = 1, \dots, n$.

The converse of the conjecture is trivial. Some authors prefer an equivalent Hankel version e.g. [7].

The conjecture originates from an older (1981) and still open conjecture from linear systems theory ([2], p. 124) on feedback cyclization over $\mathbb{C}[x]$. More background information and references can be found e.g. in [1].

To date despite several efforts ToePC is still open ([3, 4, 5, 6]). The best result till now is achieved in [7], where its truth could be established for $n \leq 8$. A proof of the truth of ToePC would at the same time be a further progress in the proof of the older conjecture

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(1) we cannot abbreviate TPC, for this denotes the twin primes conjecture.