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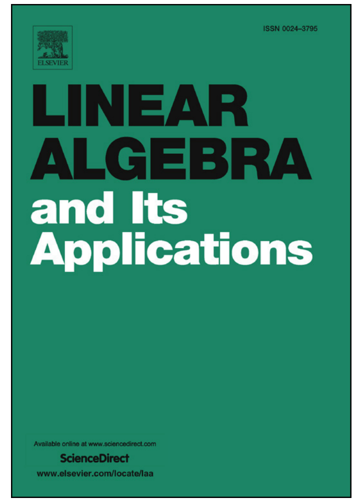

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A FORMULA TO CONSTRUCT ALL INVOLUTIONS IN RIORDAN MATRIX GROUPS.

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Abstract. We get a formula for all finite Riordan involutions. Using the natural approximation induced by the inverse limit construction, we also describe all infinite Riordan matrices which are involutions in the Riordan group. Finally, we get some consequences.

Keywords: Riordan matrices, involution, A-sequence, Babbage's equation, self-dual Riordan involutions.

MSC: 20H20, 15B99.

1. Introduction

Symmetries, in a broad sense, have always attracted scientists and in particular mathematicians. This is probably because of the natural beauty inherent to any symmetric event. Symmetries of many different kinds can be found in mathematics. For example in some numerical equalities, combinatorial identities, differential equations, group theory, topology and, of course, in geometry. Many times, driven by the natural attraction, people look for many different symmetries of the same phenomenon trying to get all of them if possible. This, naturally, leads the researcher to a greater knowledge of the phenomenon under study.

Given a group G with multiplicative notation and neutral element e, one of the most symmetric equality is given by $x^2 = e$, or more symmetrically $x^{-1} = x$. Any element in a group satisfying the previous equality is usually known as an *involution* in the group. There is an extensive literature on the study of involutions in groups. Many of these studies rely on linear representations of a group, particularly when the representation is faithful. This is because one can use linear algebra or euclidean geometry which usually is better understood than group theory. So it is important, and could have much more general consequences than it seems at a first view, to study involutions in groups of finite or infinite matrices. Again, there is an extensive literature on the study of involutions in matrix groups.

The aim of this paper is to treat involutions in a sequence of matrix groups, $\{\mathcal{R}_n\}_{n\in\mathbb{N}}$. The elements in $\mathcal{R}_n$ are lower triangular matrices of size $n+1$ with entries in a fixed field $\mathbb{K}$ of characteristic zero. For notational facts, along this paper we consider that $\mathbb{N}$ contains the integer number 0. That is, we denote by $\mathbb{N}$ the set $\{0, 1, 2, 3, \cdots\} \subset \mathbb{K}$. For every $n \in \mathbb{N}$ the