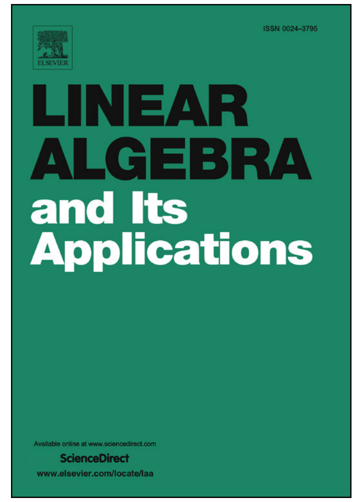

Bounds on the joint and generalized spectral radius of the Hadamard geometric mean of bounded sets of positive kernel operators

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BOUNDS ON THE JOINT AND GENERALIZED SPECTRAL RADIUS OF THE HADAMARD GEOMETRIC MEAN OF BOUNDED SETS OF POSITIVE KERNEL OPERATORS

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ABSTRACT. Let $\Psi_1, \dots \Psi_m$ be bounded sets of positive kernel operators on a Banach function space L. We prove that for the generalized spectral radius ρ and the joint spectral radius $\hat{\rho}$ the inequalities

$$\rho\left(\Psi_1^{\left(\frac{1}{m}\right)} \circ \cdots \circ \Psi_m^{\left(\frac{1}{m}\right)}\right) \le \rho(\Psi_1 \Psi_2 \cdots \Psi_m)^{\frac{1}{m}},$$

$$\hat{\rho}\left(\Psi_1^{\left(\frac{1}{m}\right)} \circ \cdots \circ \Psi_m^{\left(\frac{1}{m}\right)}\right) \le \hat{\rho}(\Psi_1 \Psi_2 \cdots \Psi_m)^{\frac{1}{m}}$$

hold, where $\Psi_1^{\left(\frac{1}{m}\right)} \circ \cdots \circ \Psi_m^{\left(\frac{1}{m}\right)}$ denotes the Hadamard (Schur) geometric mean of the sets $\Psi_1, \dots, \Psi_m$.

1. INTRODUCTION

In [34], X. Zhan conjectured that, for non-negative $n \times n$ matrices A and B, the spectral radius $\rho(A \circ B)$ of the Hadamard product satisfies

$$\rho(A \circ B) \le \rho(AB),$$

where AB denotes the usual matrix product of A and B. This conjecture was confirmed by K.M.R. Audenaert in [3] via a trace description of the spectral radius. Soon after, this inequality was reproved, generalized and refined in different ways by several authors ([18], [19], [27], [28], [26], [7], [13]). Applying a fact that the Hadamard product is a principal submatrix of the Kronecker product (i.e., by applying the technique used by R.A. Horn and F. Zhang of [18]), Z. Huang proved that

$$\rho(A_1 \circ A_2 \circ \cdots \circ A_m) \le \rho(A_1 A_2 \cdots A_m) \tag{1.1}$$

for $n \times n$ non-negative matrices $A_1, A_2, \cdots, A_m$ (see [19]). The author of the current paper extended the inequality (1.1) to non-negative matrices that define bounded operators on Banach sequence spaces in [26]. Moreover, in [26, Theorem 3.16] he generalized this inequality to the setting of the generalized and the joint spectral radius of bounded sets of such non-negative matrices. In the proofs certain results on the Hadamard product from [11] and [25] were used.

Earlier, A.R. Schep was the first one to observe that the results [11] and [25] are applicable in this context (see [27] and [28]). In particular, in [27, Theorem

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