

Accepted Manuscript

Bounds on the joint and generalized spectral radius of the Hadamard geometric mean of bounded sets of positive kernel operators

Aljoša Peperko

PII: S0024-3795(17)30440-8
DOI: <http://dx.doi.org/10.1016/j.laa.2017.07.020>
Reference: LAA 14266

To appear in: *Linear Algebra and its Applications*

Received date: 7 June 2016
Accepted date: 17 July 2017

Please cite this article in press as: A. Peperko, Bounds on the joint and generalized spectral radius of the Hadamard geometric mean of bounded sets of positive kernel operators, *Linear Algebra Appl.* (2017), <http://dx.doi.org/10.1016/j.laa.2017.07.020>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



BOUNDS ON THE JOINT AND GENERALIZED SPECTRAL RADIUS OF THE HADAMARD GEOMETRIC MEAN OF BOUNDED SETS OF POSITIVE KERNEL OPERATORS

ALJOŠA PEPERKO^{1,2}

ABSTRACT. Let Ψ_1, \dots, Ψ_m be bounded sets of positive kernel operators on a Banach function space L . We prove that for the generalized spectral radius ρ and the joint spectral radius $\hat{\rho}$ the inequalities

$$\rho\left(\Psi_1^{(\frac{1}{m})} \circ \dots \circ \Psi_m^{(\frac{1}{m})}\right) \leq \rho(\Psi_1 \Psi_2 \cdots \Psi_m)^{\frac{1}{m}},$$

$$\hat{\rho}\left(\Psi_1^{(\frac{1}{m})} \circ \dots \circ \Psi_m^{(\frac{1}{m})}\right) \leq \hat{\rho}(\Psi_1 \Psi_2 \cdots \Psi_m)^{\frac{1}{m}}$$

hold, where $\Psi_1^{(\frac{1}{m})} \circ \dots \circ \Psi_m^{(\frac{1}{m})}$ denotes the Hadamard (Schur) geometric mean of the sets Ψ_1, \dots, Ψ_m .

1. INTRODUCTION

In [34], X. Zhan conjectured that, for non-negative $n \times n$ matrices A and B , the spectral radius $\rho(A \circ B)$ of the Hadamard product satisfies

$$\rho(A \circ B) \leq \rho(AB),$$

where AB denotes the usual matrix product of A and B . This conjecture was confirmed by K.M.R. Audenaert in [3] via a trace description of the spectral radius. Soon after, this inequality was reproved, generalized and refined in different ways by several authors ([18], [19], [27], [28], [26], [7], [13]). Applying a fact that the Hadamard product is a principal submatrix of the Kronecker product (i.e., by applying the technique used by R.A. Horn and F. Zhang of [18]), Z. Huang proved that

$$\rho(A_1 \circ A_2 \circ \dots \circ A_m) \leq \rho(A_1 A_2 \cdots A_m) \tag{1.1}$$

for $n \times n$ non-negative matrices A_1, A_2, \dots, A_m (see [19]). The author of the current paper extended the inequality (1.1) to non-negative matrices that define bounded operators on Banach sequence spaces in [26]. Moreover, in [26, Theorem 3.16] he generalized this inequality to the setting of the generalized and the joint spectral radius of bounded sets of such non-negative matrices. In the proofs certain results on the Hadamard product from [11] and [25] were used.

Earlier, A.R. Schep was the first one to observe that the results [11] and [25] are applicable in this context (see [27] and [28]). In particular, in [27, Theorem

2010 *Mathematics Subject Classification.* 15A42, 15A60, 47B65, 47B34, 47A10, 15B48.

Key words and phrases. Hadamard-Schur geometric mean; Hadamard-Schur product; joint and generalized spectral radius; positive kernel operators; non-negative matrices; bounded sets of operators.

Download English Version:

<https://daneshyari.com/en/article/5773016>

Download Persian Version:

<https://daneshyari.com/article/5773016>

[Daneshyari.com](https://daneshyari.com)