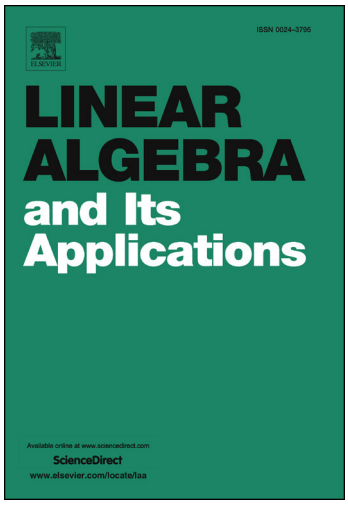

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Trace inequalities for positive semidefinite block matrices

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Abstract

If a 2×2 block matrix $\begin{bmatrix} A & B \\ B^* & C \end{bmatrix}$ is positive semidefinite, where each block is square, then the following trace inequality holds

$$|\mathrm{tr}AC - \mathrm{tr}B^*B| \leq \mathrm{tr}A\mathrm{tr}C - |\mathrm{tr}B|^2.$$

This improves a result of Besenyei [2]. Moreover, we show that

$$|\mathrm{tr}B^m| \leq \mathrm{tr}A^{m/2}C^{m/2}, \qquad m = 2, 3, \ldots.$$

Keywords: trace inequality, majorization, eigenvalue.

2010 MSC: 15A45, 15A42

1. Introduction

In the IMAGE problem 50-3, Besenyei [2] drew us attention to the following novel trace inequality: If $\begin{bmatrix} A & B \\ B^* & C \end{bmatrix}$ is positive semidefinite with each block square, then

$$\mathrm{tr}AC - \mathrm{tr}B^*B \leq \mathrm{tr}A\mathrm{tr}C - |\mathrm{tr}B|^2. \tag{1}$$

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