Linear Algebra and its Applications 523 (2017) 33–45

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

The maximum spectral radii of uniform supertrees with given degree sequences ☆

Peng Xiao [a,b], Ligong Wang [a,*], Yong Lu [a]

[a] Department of Applied Mathematics, School of Science, Northwestern Polytechnical University, Xi'an 710072, PR China
[b] College of Arts and Sciences, Shaanxi University of Science and Technology, Xi'an 710021, PR China

ARTICLE INFO

Article history:
Received 13 June 2016
Accepted 13 February 2017
Available online 20 February 2017
Submitted by R. Brualdi

MSC:
05C50
15A42

Keywords:
Spectral radius
Supertree
Degree sequence
Majorization

ABSTRACT

This paper generalizes the breadth-first-search ordering on trees to k-uniform supertrees. By using 2-switch operation and edge-moving operation on supertrees, we determine the unique k-uniform supertree with the maximum spectral radius among all k-uniform supertrees with a given degree sequence. Moreover, we generalize the majorization theorem on trees to k-uniform supertrees.

1. Introduction

We denote the set $\{1, 2, \dots, n\}$ by $[n]$. Let $G = (V(G), E(G))$ be a hypergraph with vertex set $V(G) = \{1, 2, \dots, n\}$ and edge set $E(G) = \{e_1, e_2, \dots, e_m\}$, where $e_i = \{i_1, i_2, \dots, i_l\}, i_j \in [n]$, $j = 1, 2, \dots, l$. If $|e_i| = k$ for any $i = 1, 2, \dots, m$, then

☆ Supported by the National Natural Science Foundation of China (No. 11171273).

* Corresponding author.
E-mail addresses: xiaopeng@sust.edu.cn (P. Xiao), lgwangmath@163.com (L. Wang), luyong.gougou@163.com (Y. Lu).

http://dx.doi.org/10.1016/j.laa.2017.02.018
0024-3795/

G is called a k-uniform hypergraph. Two vertices in a hypergraph are adjacent if there is an edge which contains both vertices. A vertex v is said to be incident to an edge e if $v \in e$. We say that G is without multiple edge, if $e_i \neq e_j$ if and only if $i \neq j$, $1 \leq i, j \leq m$.

Let $G = (V, E)$ be a hypergraph, a path P in G from u to v be a vertex-edge alternative sequence: $u = u_1, e_1, u_2, e_2, \ldots, u_s, e_s, u_{s+1} = v$ such that (a) $u_1, u_2, \ldots, u_{s+1}$ are distinct vertices; (b) $e_1, e_2, \ldots, e_s$ are distinct edges; (c) $u_i, u_{i+1} \in e_i$ for $i = 1, 2, \ldots, s$. If $u = v$, then the path P is called a cycle. A hypergraph is connected if for any pair of vertices, there is a path which connects these vertices; it is not connected otherwise. The integer s is the length of the path P. The distance $d(u, v)$ between two vertices u and v is the minimum length of a path which connects u and v. The diameter $d(G)$ of G is defined by $d(G) = \max\{d(u, v) : u, v \in V\}$. For a vertex $v \in V$, the degree of v (denoted by d_v or $d(v)$) is the number of edges of G that contain v. A vertex of degree one is called a pendent vertex. The list of degrees of vertices in a hypergraph G is called the degree sequence of G. These definitions and notations can be found in [1,9].

Definition 1.1. ([1]) Let $G = (V, E)$ and $G' = (V', E')$ be two hypergraphs without multiple edge. We say that G is isomorphic to G', written by $G \cong G'$, if there exists a bijection $f : V \to V'$ such that $\forall\ e = \{u_1, u_2, \ldots, u_k\} : e \in E \Leftrightarrow f(e) = \{f(u_1), f(u_2), \ldots, f(u_k)\} \in E'$.

Li, Shao and Qi [9] introduced the concept of supertrees.

Definition 1.2. ([9]) A supertree is a hypergraph which is both connected and acyclic.

Lemma 1.1. *([9]) A connected k-uniform hypergraph with n vertices and m edges is acyclic if and only if $m = \frac{n-1}{k-1}$.*

From the definition of supertrees, we can see that each pair of the edges of a supertree shares at most one common vertex.

Since the eigenvalues of higher-order tensors were independently proposed by Qi [11] and Lim [8], numerous contributions to a framework for understanding the spectra of k-uniform hypergraphs via tensors have appeared. There is a rich and more general theory of eigenvalues for tensors (see [6,12,14]).

Definition 1.3. An order k dimension n tensor $\mathcal{A} = (a_{i_1 i_2 \ldots i_k}) \in \mathbb{C}^{n \times n \times \ldots \times n}$ is a multidimensional array with all entries $a_{i_1 i_2 \ldots i_k} \in \mathbb{C}$ $(i_1, i_2, \ldots, i_k \in [n])$.

A tensor $\mathcal{A}$ is said to be symmetric, if its entries are invariant under any permutation of their indices.

Definition 1.4. Let $G = (V, E)$ be a k-uniform hypergraph on n vertices. The adjacency tensor $\mathcal{A}$ of G is defined as the order k dimension n tensor $\mathcal{A}(G)$ whose $(i_1 \ldots i_k)$-entry is: