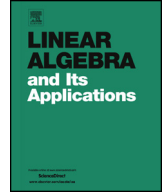




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# The maximum spectral radii of uniform supertrees with given degree sequences <sup>☆</sup>



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## ABSTRACT

This paper generalizes the breadth-first-search ordering on trees to  $k$ -uniform supertrees. By using 2-switch operation and edge-moving operation on supertrees, we determine the unique  $k$ -uniform supertree with the maximum spectral radius among all  $k$ -uniform supertrees with a given degree sequence. Moreover, we generalize the majorization theorem on trees to  $k$ -uniform supertrees.

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## 1. Introduction

We denote the set  $\{1, 2, \dots, n\}$  by  $[n]$ . Let  $G = (V(G), E(G))$  be a hypergraph with vertex set  $V(G) = \{1, 2, \dots, n\}$  and edge set  $E(G) = \{e_1, e_2, \dots, e_m\}$ , where  $e_i = \{i_1, i_2, \dots, i_l\}$ ,  $i_j \in [n]$ ,  $j = 1, 2, \dots, l$ . If  $|e_i| = k$  for any  $i = 1, 2, \dots, m$ , then

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$G$  is called a  $k$ -uniform hypergraph. Two vertices in a hypergraph are adjacent if there is an edge which contains both vertices. A vertex  $v$  is said to be incident to an edge  $e$  if  $v \in e$ . We say that  $G$  is without multiple edge, if  $e_i \neq e_j$  if and only if  $i \neq j, 1 \leq i, j \leq m$ .

Let  $G = (V, E)$  be a hypergraph, a path  $P$  in  $G$  from  $u$  to  $v$  be a vertex-edge alternative sequence:  $u = u_1, e_1, u_2, e_2, \dots, u_s, e_s, u_{s+1} = v$  such that (a)  $u_1, u_2, \dots, u_{s+1}$  are distinct vertices; (b)  $e_1, e_2, \dots, e_s$  are distinct edges; (c)  $u_i, u_{i+1} \in e_i$  for  $i = 1, 2, \dots, s$ . If  $u = v$ , then the path  $P$  is called a cycle. A hypergraph is connected if for any pair of vertices, there is a path which connects these vertices; it is not connected otherwise. The integer  $s$  is the length of the path  $P$ . The distance  $d(u, v)$  between two vertices  $u$  and  $v$  is the minimum length of a path which connects  $u$  and  $v$ . The diameter  $d(G)$  of  $G$  is defined by  $d(G) = \max\{d(u, v) : u, v \in V\}$ . For a vertex  $v \in V$ , the degree of  $v$  (denoted by  $d_v$  or  $d(v)$ ) is the number of edges of  $G$  that contain  $v$ . A vertex of degree one is called a pendent vertex. The list of degrees of vertices in a hypergraph  $G$  is called the degree sequence of  $G$ . These definitions and notations can be found in [1,9].

**Definition 1.1.** ([1]) Let  $G = (V, E)$  and  $G' = (V', E')$  be two hypergraphs without multiple edge. We say that  $G$  is isomorphic to  $G'$ , written by  $G \cong G'$ , if there exists a bijection  $f : V \rightarrow V'$  such that  $\forall e = \{u_1, u_2, \dots, u_k\} : e \in E \Leftrightarrow f(e) = \{f(u_1), f(u_2), \dots, f(u_k)\} \in E'$ .

Li, Shao and Qi [9] introduced the concept of supertrees.

**Definition 1.2.** ([9]) A supertree is a hypergraph which is both connected and acyclic.

**Lemma 1.1.** ([9]) A connected  $k$ -uniform hypergraph with  $n$  vertices and  $m$  edges is acyclic if and only if  $m = \frac{n-1}{k-1}$ .

From the definition of supertrees, we can see that each pair of the edges of a supertree shares at most one common vertex.

Since the eigenvalues of higher-order tensors were independently proposed by Qi [11] and Lim [8], numerous contributions to a framework for understanding the spectra of  $k$ -uniform hypergraphs via tensors have appeared. There is a rich and more general theory of eigenvalues for tensors (see [6,12,14]).

**Definition 1.3.** An order  $k$  dimension  $n$  tensor  $\mathcal{A} = (a_{i_1 i_2 \dots i_k}) \in \mathbb{C}^{n \times n \times \dots \times n}$  is a multidimensional array with all entries  $a_{i_1 i_2 \dots i_k} \in \mathbb{C}$  ( $i_1, i_2, \dots, i_k \in [n]$ ).

A tensor  $\mathcal{A}$  is said to be symmetric, if its entries are invariant under any permutation of their indices.

**Definition 1.4.** Let  $G = (V, E)$  be a  $k$ -uniform hypergraph on  $n$  vertices. The adjacency tensor  $\mathcal{A}$  of  $G$  is defined as the order  $k$  dimension  $n$  tensor  $\mathcal{A}(G)$  whose  $(i_1 \dots i_k)$ -entry is:

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