

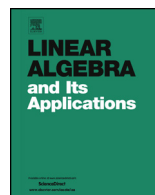


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Dilations, wandering subspaces, and inner functions



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ABSTRACT

The objective of this paper is to study wandering subspaces for commuting tuples of bounded operators on Hilbert spaces. It is shown that, for a large class of analytic functional Hilbert spaces \mathcal{H}_K on the unit ball in \mathbb{C}^n , wandering subspaces for restrictions of the multiplication tuple $M_z = (M_{z_1}, \dots, M_{z_n})$ can be described in terms of suitable \mathcal{H}_K -inner functions. We prove that \mathcal{H}_K -inner functions are contractive multipliers and deduce a result on the multiplier norm of quasi-homogeneous polynomials as an application. Along the way we prove a refinement of a result of Arveson on the uniqueness of minimal dilations of pure row contractions.

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1. Introduction

Let $T = (T_1, \dots, T_n)$ be an n -tuple of commuting bounded linear operators on a complex Hilbert space \mathcal{H} . A closed subspace $\mathcal{W} \subset \mathcal{H}$ is called a *wandering subspace* for T if

$$\mathcal{W} \perp T^{\mathbf{k}}\mathcal{W} \quad (\mathbf{k} \in \mathbb{N}^n \setminus \{\mathbf{0}\}).$$

We say that \mathcal{W} is a *generating wandering subspace* for T if in addition

$$\mathcal{H} = \overline{\text{span}}\{T^{\mathbf{k}}\mathcal{W} : \mathbf{k} \in \mathbb{N}^n\}.$$

Wandering subspaces were defined by Halmos in [10]. One of the main observations from [10] is the following. Let \mathcal{E} be a Hilbert space and let $M_z : H^2_{\mathcal{E}}(\mathbb{D}) \rightarrow H^2_{\mathcal{E}}(\mathbb{D})$ be the operator of multiplication with the argument on the \mathcal{E} -valued Hardy space $H^2_{\mathcal{E}}(\mathbb{D})$ on the unit disc \mathbb{D} . Suppose that \mathcal{S} is a non-trivial closed M_z -invariant subspace of $H^2_{\mathcal{E}}(\mathbb{D})$. Then

$$\mathcal{W} = \mathcal{S} \ominus z\mathcal{S}$$

is a wandering subspace for $M_z|_{\mathcal{S}}$ such that

$$M_z^p\mathcal{W} \perp M_z^q\mathcal{W}$$

for all $p \neq q$ in \mathbb{N} and

$$\mathcal{S} = \overline{\text{span}}\{z^m\mathcal{W} : m \in \mathbb{N}\}.$$

Hence

$$\mathcal{S} = \bigoplus_{m=0}^{\infty} z^m\mathcal{W}$$

and up to unitary equivalence

$$M_z|_{\mathcal{S}} \text{ on } \mathcal{S} \cong M_z \text{ on } H^2_{\mathcal{W}}(\mathbb{D}).$$

In particular, we have $\mathcal{S} = V(H^2_{\mathcal{W}}(\mathbb{D}))$, where $V : H^2_{\mathcal{W}}(\mathbb{D}) \rightarrow H^2_{\mathcal{E}}(\mathbb{D})$ is an isometry and $VM_z = M_zV$. One can show (see Lemma V.3.2 in [14] for details and more precise references) that any such intertwining isometry V acts as the multiplication operator $V = M_{\Theta} : H^2_{\mathcal{W}}(\mathbb{D}) \rightarrow H^2_{\mathcal{E}}(\mathbb{D})$, $f \mapsto \Theta f$, with a bounded analytic function $\Theta \in H^{\infty}_{\mathcal{B}(\mathcal{W}, \mathcal{E})}(\mathbb{D})$ such that Θ possesses isometric boundary values almost everywhere. In this case

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