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Partial eigenvalue assignment with time delay in high order system using the receptance

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ABSTRACT

An explicit solution to the partial eigenvalue assignment problem of high order control system is presented by the method of receptance. Conventional methods, e.g. finite elements, are known to contain inaccuracies and assumptions that may hinder the calculations. An alternative approach was given by Ram and Mottershead [1] in the form of receptances, typically available from a modal test. This paper generalizes the earlier work on partial assignment that is applicable to multi-input delayed system without use of the Sherman–Morrison formula. The results of our numerical experiments support the validity of our proposed method.

1. Introduction

Consider the high order dynamical system

$$P_K(D)v(t) = f(t), \tag{1.1}$$

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where $P_K(D) = \sum_{k=0}^{K} M_k D^k$, $D^k = \frac{\mathrm{d}^k}{\mathrm{d}t^k}$ is a differential operator, M_k is real constant $n \times n$ matrices, $k = 0, 1, \ldots, K$, and M_K is nonsingular. Using separation of variables of the form $v(t) = x\mathrm{e}^{\lambda t}$ in $P_K(D)v(t) = 0$, where x is a constant vector, we obtain the high order eigenvalue problem

$$P_K(\lambda)x = 0, \tag{1.2}$$

where

$$P_K(\lambda) = \sum_{k=0}^{K} M_k \lambda^k. \tag{1.3}$$

The characteristic roots $\lambda_1, \ldots, \lambda_{Kn}$ of the polynomial Eq. (1.3), namely $\det[P_K(\lambda_i)] = 0$, $i = 1, 2, \ldots, Kn$, are known as eigenvalues or natural frequencies. The nonzero corresponding vectors $x_1, \ldots, x_{Kn}$ are corresponding right eigenvectors or mode shapes which satisfy respectively

$$P_K(\lambda_i)x_i = 0, \ i = 1, 2, \ldots, Kn. \tag{1.4}$$

From the knowledge of these, the system responses as well as stability can be determined. It is well known that if the Kn eigenvalues $\{\lambda_i\}_{i=1}^{Kn}$ of Eq. (1.3) satisfy $\mathrm{Re}(\lambda_i) \leq 0$ for all $i = 1, 2, \ldots, Kn$, then the response of Eq. (1.1) is bounded for arbitrary initial conditions. The response of the system to initial conditions is required in some applications to diminish rapidly. This objective can be achieved by assigning eigenvalues further to the left-hand side of the complex plane.

In most practical applications only a small number of eigenvalues, which are responsible for instability and other undesirable phenomenons, need to be assigned. Active control strategy is an effective way to control dangerous poles in a structure. Implementation of this strategy requires real-time computations of feedback control matrices such that a small amount of eigenvalues of the associated matrix pencil are replaced by suitably chosen ones while the remaining large number of eigenvalues and eigenvectors remain unchanged ensuring the no spill-over. This consideration gives rise to partial eigenvalue assignment problem (PEAP).

Orthogonality conditions given by Fawzy and Bishop [2] were used to derive an explicit solution to the partial eigenvalue assignment problem. The solution shed new light on the stabilization and control of large flexible space structures. Since the introduction of PEAP in the literature by Datta, Elhay and Ram [3] showed an elegant mathematical solution to the single input case, much research has been done in recent years. These works have extended the single input solutions to the multi-input case [4–6] and also to the solution of eigenstructure assignment problem [7]. Furthermore, the aspects of robust feedback stabilization and the norm minimization have been considered and solved by the sophisticated techniques of numerical linear algebra and numerical optimization [8,9].