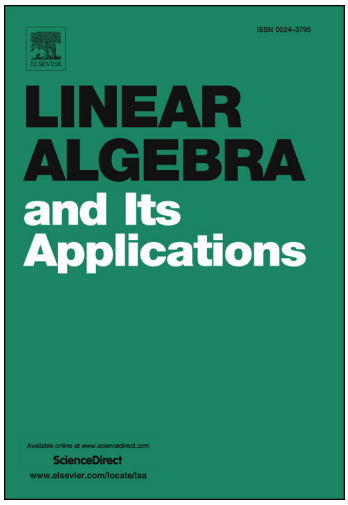


# The unconditional constants for Hilbert space frame expansions

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# THE UNCONDITIONAL CONSTANTS FOR HILBERT SPACE FRAME EXPANSIONS

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Abstract. The most fundamental notion in frame theory is the frame expansion of a vector. Although it is well known that these expansions are unconditionally convergent series, no characterizations of the unconditional constant were known. This has made it impossible to get accurate quantitative estimates for problems which require using subsequences of a frame. We will prove some new results in frame theory by showing that the unconditional constants of the frame expansion of a vector in a Hilbert space are bounded by $\sqrt{\frac{B}{A}}$, where $A, B$ are the frame bounds of the frame. Tight frames thus have unconditional constant one, which we then generalize by showing that Bessel sequences have frame expansions with unconditional constant one if and only if the sequence is an orthogonal sum of tight frames. We give further results concerning frame expansions, in which we examine when $\sqrt{\frac{B}{A}}$ is actually attained or not. We end by discussing the connections of this work to *frame multipliers*. These results hold in both real and complex Hilbert spaces.

**Keywords** Hilbert space frames; Frame operator; Frame expansion; Unconditional convergence

**AMS Classification** 42C15

## 1. Introduction

Hilbert space frames have traditionally been used in signal processing. But over the last few years, frame theory has become one of the most applied subjects in mathematics. Fundamental to the notion of a Bessel sequence, and more specifically a frame, is that it is a *possibly*

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