Linear Algebra and its Applications 521 (2017) 70–103

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On bipartite unitary matrices generating subalgebra-preserving quantum operations

Tristan Benoist, Ion Nechita *

CNRS, Laboratoire de Physique Théorique, Toulouse, France

A R T I C L E I N F O

Article history:
Received 1 September 2016
Accepted 16 January 2017
Submitted by C.-K. Li

MSC:
15B48
81P45

Keywords:
Quantum channel
Completely positive map
Invariant sub-algebra
Quantum Latin square
Sinkhorn algorithm
Operator scaling

A B S T R A C T

We study the structure of bipartite unitary operators which generate via the Stinespring dilation theorem, quantum operations preserving some given matrix algebra, independently of the ancilla state. We characterize completely the unitary operators preserving diagonal, block-diagonal, and tensor product algebras. Some unexpected connections with the theory of quantum Latin squares are explored, and we introduce and study a Sinkhorn-like algorithm used to randomly generate quantum Latin squares.

1. Introduction

States of finite dimensional quantum systems are described by trace one, positive semidefinite matrices called *density matrices*. Their evolution is given by completely positive trace preserving maps (CPTP), also called *quantum channels*. For *closed* systems,

* Corresponding author.
E-mail addresses: tristan.benoist@irsamc.ups-tlse.fr (T. Benoist), nechita@irsamc.ups-tlse.fr (I. Nechita).

http://dx.doi.org/10.1016/j.laa.2017.01.020

the Quantum channel consists in the left and right multiplication by a unitary matrix and its hermitian conjugate respectively. Quantum channels describing the evolution of a system in contact with an environment, the so-called *open* quantum systems, are given by more general CPTP maps. The famous Stinespring dilation theorem [20] connects the mathematical definition of quantum channels with their physical interpretation: the evolution of an open quantum system can be seen as the closed evolution of a [system + environment] bipartite system, followed by the discarding (or partial tracing) of the environment:

$$\rho \mapsto T(\rho) = [\mathrm{id} \otimes \mathrm{Tr}](U(\rho \otimes \beta)U^*).$$

Hence, the quantum evolution depends on two physical relevant parameters: the global evolution operator U and the state of the environment subsystem β. We would like to identify the part played by the unitary interaction U in the properties of the quantum channel T. Namely, we would like to characterize families of bipartite unitary matrices U such that the quantum channels T they generate have some prescribed properties, independently of the environment state β. In this work, we answer this question for channels preserving some special sub-algebras of states/observables. Different channel properties were studied under the same framework in [11] (see also [4]).

Part of the initial motivation of our study originates in the characterization of quantum channels preserving a set of *pointer* states. These CPTP maps appear in the context of quantum non demolition measurements [6–8,2,22,21]. The following proposition is a discrete time version of [8, Theorem 3]. It is obtained through the repeated application of Theorem 7.1.

Proposition 1.1. *Let $\{e_i\}_{i=1}^n$ be a fixed orthonormal basis of $\mathbb{C}^n$, and $U \in \mathcal{U}_{nk}$ a bipartite unitary operator. The following statements are equivalent:*

(1) *There exists a set of quantum states $\mathcal{B}$ which spans $M_k(\mathbb{C})$ such that, for any state $\beta \in \mathcal{B}$, the quantum channel $T_{U,\beta}$ leaves invariant the basis states $e_i e_i^*$:*

$$\forall \beta \in \mathcal{B}, \forall i \in [n], \qquad T_{U,\beta}(e_i e_i^*) = e_i e_i^*;$$

(2) *The operator U is block-diagonal, i.e. there exists unitary operators $U_1, \ldots, U_n \in \mathcal{U}_k$ such that*

$$U = \sum_{i=1}^{n} e_i e_i^* \otimes U_i.$$

The implication (1) $\Longrightarrow$ (2) holds also if (1) is replaced by

$$\exists \beta > 0, \forall i \in [n], \qquad T_{U,\beta}(e_i e_i^*) = e_i e_i^*.$$