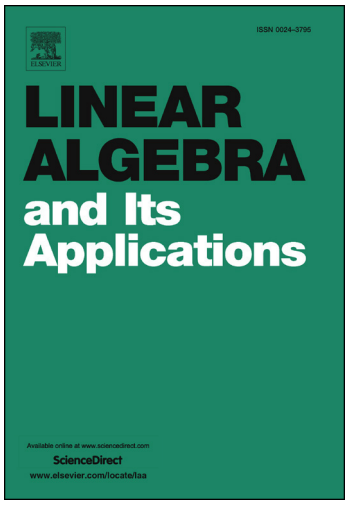

Vector bundles give equations of cactus varieties

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Vector bundles give equations of cactus varieties

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Abstract

We prove that vector bundles give equations of cactus varieties. We derive from it that equations coming from vector bundles are not enough to define secant varieties of Veronese varieties in general.

Keywords: secant variety, Waring rank, cactus rank, homogeneous variety, apolarity, catalecticant
2010 MSC: 14N15, 14M17

1. Introduction

Suppose W is a vector space over an algebraically closed field $\mathbb{K}$. We denote by W^* the dual vector space. Let $X \subseteq \mathbb{P}W$ be a non-degenerate (i.e. not contained in a hyperplane) projective variety over $\mathbb{K}$. We assume all the sheaves considered on X are coherent $\mathcal{O}_X$-modules. For $F \in W$ let us define the notion of X-rank.

$$\mathrm{r}_X(F) = \min\{r \in \mathbb{Z}_{\geq 0} | [F] \in \langle p_1, \ldots, p_r \rangle \text{ for some } p_1, \ldots, p_r \in X\},$$

where $[F]$ denotes the class of F in the projective space, and $\langle \cdot \rangle$ denotes the (projective) linear span. The r-th secant variety is

$$\begin{aligned}\sigma_r(X) &= \overline{\{[F] \in \mathbb{P}W | \, \mathrm{r}_X(F) \leq r\}} \\ &= \overline{\bigcup_{p_1,\ldots,p_r \in X} \langle p_1, \ldots, p_r \rangle},\end{aligned}$$

where the overline denotes the Zariski closure. The variety X is often fixed, so we omit X in the X-rank and $\mathrm{r}_X(F)$, and simply write rank and $\mathrm{r}(F)$.

In this article we investigate the problem of finding set-theoretic equations of $\sigma_r(X)$ and the problem of giving lower bounds for rank. The following proposition, which is given for instance in [1, beginning of Chapter 7], is useful:

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