

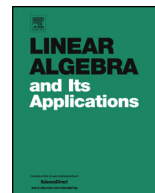


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## An explicit description of the irreducible components of the set of matrix pencils with bounded normal rank <sup>☆</sup>



Fernando De Terán <sup>a,\*</sup>, Froilán M. Dopico <sup>a</sup>, J.M. Landsberg <sup>b</sup>

<sup>a</sup> *Departamento de Matemáticas, Universidad Carlos III de Madrid, Avda. Universidad 30, 28911 Leganés, Spain*

<sup>b</sup> *Department of Mathematics, Texas A&M University, Mailstop 3368 College Station, TX 77843-3368, United States*

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### ABSTRACT

The set of  $m \times n$  singular matrix pencils with normal rank at most  $r$  is an algebraic set with  $r + 1$  irreducible components. These components are the closure of the orbits (under strict equivalence) of  $r + 1$  matrix pencils which are in Kronecker canonical form. In this paper, we provide a new explicit description of each of these irreducible components which is a parametrization of each component. Therefore one can explicitly construct any pencil in each of these components. The new description of each of these irreducible components consists of the sum of  $r$  rank-1 matrix pencils, namely, a column polynomial vector of degree at most 1 times a row polynomial vector of degree at most 1, where we impose one of these two vectors to have degree zero. The number

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\* Corresponding author.

*E-mail addresses:* [fteran@math.uc3m.es](mailto:fteran@math.uc3m.es) (F. De Terán), [dopico@math.uc3m.es](mailto:dopico@math.uc3m.es) (F.M. Dopico), [jml@math.tamu.edu](mailto:jml@math.tamu.edu) (J.M. Landsberg).

of row vectors with zero degree determines each irreducible component.

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## 1. Introduction

We are concerned in this paper with *singular* matrix pencils  $A + \lambda B$ , with  $A, B \in \mathbb{C}^{m \times n}$ . This includes rectangular pencils ( $m \neq n$ ) and square ones ( $m = n$ ) with  $\det(A + \lambda B)$  identically zero as a polynomial in  $\lambda$ . More precisely, our interest focuses on the set  $\mathcal{P}_r^{m \times n}$  of  $m \times n$  matrix pencils with complex coefficients and normal rank at most  $r$ , with  $r < \min\{m, n\}$ .

In the contexts where matrix pencils usually arise, e.g., systems of first order ordinary differential equations with constant coefficients  $Ax + Bx' = f(t)$ , the relevant information is encoded in the *Kronecker canonical form* of the pencil (in the following, KCF, or  $\text{KCF}(A + \lambda B)$  when it refers to a particular pencil). This is the canonical form under strict equivalence of matrix pencils (see Section 2). The computation of the KCF of a given pencil  $A + \lambda B$  is a delicate task, because it is not a continuous function of the entries of  $A$  and  $B$  (see, e.g., [2]). Nonetheless, when a good algorithm (for instance, the backward stable one in [19]) is used to compute the KCF, the output is the KCF of a pencil  $\tilde{A} + \lambda \tilde{B}$ , “nearby” to the exact one, more precisely, a KCF that contains the exact KCF in its orbit closure, as explained in the next paragraph. In this setting, the analysis of the geometry of the set of  $m \times n$  matrix pencils may be useful [10,11]. In particular, the knowledge of all KCFs of the pencils included in the orbit closure of a given KCF could improve our understanding of possible failures of the algorithms, and to develop enhanced versions of these algorithms.

Two  $m \times n$  matrix pencils  $Q_1(\lambda)$  and  $Q_2(\lambda)$  are said to be *strictly equivalent* if there exist two constant nonsingular matrices  $E \in \mathbb{C}^{m \times m}$  and  $F \in \mathbb{C}^{n \times n}$  such that  $EQ_1(\lambda)F = Q_2(\lambda)$ . We identify each orbit under strict equivalence with the KCF of any pencil in this orbit (by definition, they all have the same KCF). Then we say that some KCF,  $K_1 + \lambda K_2$ , degenerates to the KCF  $\tilde{K}_1 + \lambda \tilde{K}_2$  if  $\tilde{K}_1 + \lambda \tilde{K}_2$  belongs to the closure of the orbit of  $K_1 + \lambda K_2$ . In other words, if there is a sequence of matrix pencils,  $A_m + \lambda B_m$ , all having the same KCF, namely  $K_1 + \lambda K_2$ , which converges to a pencil whose KCF is  $\tilde{K}_1 + \lambda \tilde{K}_2$ . There are some cases where it is easy to determine, even at a first glance, whether a given KCF degenerates to some other one or not. This happens, for instance, with the following two pencils in KCF:

$$K(\lambda) = \begin{pmatrix} \lambda & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \tilde{K}(\lambda) = \begin{pmatrix} \lambda & 0 \\ 0 & 0 \end{pmatrix}.$$

It holds that  $K(\lambda)$  degenerates to  $\tilde{K}(\lambda)$ , since the sequence  $\{K^{(m)}(\lambda)\}_{m \in \mathbb{N}}$ , with

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