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An explicit description of the irreducible components of the set of matrix pencils with bounded normal rank ☆

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The set of $m \times n$ singular matrix pencils with normal rank at most r is an algebraic set with $r + 1$ irreducible components. These components are the closure of the orbits (under strict equivalence) of $r + 1$ matrix pencils which are in Kronecker canonical form. In this paper, we provide a new explicit description of each of these irreducible components which is a parametrization of each component. Therefore one can explicitly construct any pencil in each of these components. The new description of each of these irreducible components consists of the sum of r rank-1 matrix pencils, namely, a column polynomial vector of degree at most 1 times a row polynomial vector of degree at most 1, where we impose one of these two vectors to have degree zero. The number

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of row vectors with zero degree determines each irreducible component.

1. Introduction

We are concerned in this paper with *singular* matrix pencils $A + \lambda B$, with $A, B \in \mathbb{C}^{m\times n}$. This includes rectangular pencils ($m \neq n$) and square ones ($m = n$) with $\det(A+\lambda B)$ identically zero as a polynomial in λ. More precisely, our interest focuses on the set $\mathcal{P}_r^{m\times n}$ of $m \times n$ matrix pencils with complex coefficients and normal rank at most r, with $r < \min\{m, n\}$.

In the contexts where matrix pencils usually arise, e.g., systems of first order ordinary differential equations with constant coefficients $Ax+Bx' = f(t)$, the relevant information is encoded in the *Kronecker canonical form* of the pencil (in the following, KCF, or KCF$(A + \lambda B)$ when it refers to a particular pencil). This is the canonical form under strict equivalence of matrix pencils (see Section 2). The computation of the KCF of a given pencil $A + \lambda B$ is a delicate task, because it is not a continuous function of the entries of A and B (see, e.g., [2]). Nonetheless, when a good algorithm (for instance, the backward stable one in [19]) is used to compute the KCF, the output is the KCF of a pencil $\widetilde{A}+\lambda\widetilde{B}$, "nearby" to the exact one, more precisely, a KCF that contains the exact KCF in its orbit closure, as explained in the next paragraph. In this setting, the analysis of the geometry of the set of $m \times n$ matrix pencils may be useful [10,11]. In particular, the knowledge of all KCFs of the pencils included in the orbit closure of a given KCF could improve our understanding of possible failures of the algorithms, and to develop enhanced versions of these algorithms.

Two $m \times n$ matrix pencils $Q_1(\lambda)$ and $Q_2(\lambda)$ are said to be *strictly equivalent* if there exist two constant nonsingular matrices $E \in \mathbb{C}^{m\times m}$ and $F \in \mathbb{C}^{n\times n}$ such that $EQ_1(\lambda)F = Q_2(\lambda)$. We identify each orbit under strict equivalence with the KCF of any pencil in this orbit (by definition, they all have the same KCF). Then we say that some KCF, $K_1 + \lambda K_2$, degenerates to the KCF $\widetilde{K}_1 + \lambda\widetilde{K}_2$ if $\widetilde{K}_1 + \lambda\widetilde{K}_2$ belongs to the closure of the orbit of $K_1 + \lambda K_2$. In other words, if there is a sequence of matrix pencils, $A_m + \lambda B_m$, all having the same KCF, namely $K_1 + \lambda K_2$, which converges to a pencil whose KCF is $\widetilde{K}_1 + \lambda\widetilde{K}_2$. There are some cases where it is easy to determine, even at a first glance, whether a given KCF degenerates to some other one or not. This happens, for instance, with the following two pencils in KCF:

$$K(\lambda) = \begin{pmatrix} \lambda & 1 \\ 0 & 0 \end{pmatrix} \qquad \text{and} \qquad \widetilde{K}(\lambda) = \begin{pmatrix} \lambda & 0 \\ 0 & 0 \end{pmatrix}.$$

It holds that $K(\lambda)$ degenerates to $\widetilde{K}(\lambda)$, since the sequence $\left\{K^{(m)}(\lambda)\right\}_{m\in\mathbb{N}}$, with