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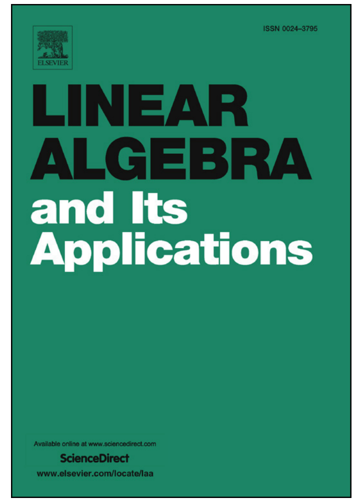

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Multigrid methods: grid transfer operators and subdivision schemes

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Abstract

The convergence rate of a multigrid method depends on the properties of the smoother and the so-called grid transfer operator. In this paper we define and analyze new grid transfer operators with a generic cutting size which are applicable for high order problems. We enlarge the class of available geometric grid transfer operators by relating the symbol analysis of the coarse grid correction with the approximation properties of univariate subdivision schemes. We show that the polynomial generation property and stability of a subdivision scheme are crucial for convergence and optimality of the corresponding multigrid method. We construct a new class of grid transfer operators from univariate primal binary and ternary pseudo-spline symbols. Our numerical results illustrate the behavior of the new grid transfer operators and provide promising preliminary results for the bivariate case.

1. Introduction

Our main goal is to establish a link between subdivision, efficient iterative methods used in geometric data processing, and multigrid methods, fast iterative solvers for sparse and ill-conditioned linear systems. For transparency of exposition, we present our results for the case of univariate primal subdivision schemes and vertex-centered discretizations of one dimensional elliptic problems. Nevertheless, some preliminary numerical results are provided also for the bivariate case.

Multigrid methods are used mainly for solving linear systems of equations

$$A_n \mathbf{x} = \mathbf{b}_n, \quad \mathbf{x} \in \mathbb{C}^n, \tag{1.1}$$

with symmetric and positive definite system matrices $A_n \in \mathbb{C}^{n\times n}$ and $\mathbf{b}_n \in \mathbb{C}^n$, $n \in \mathbb{N}$. A basic two-grid method combines the action of a smoother and a coarse grid correction: the smoother is often a simple iterative method such as Gauss-Seidel; the coarse grid correction amounts to solving the residual equation exactly on a coarser grid. A V-cycle multigrid method solves the residual equation approximately within the recursive application of the two-grid method, until the coarsest level is reached and there the resulting small system of equations is solved exactly [8, 34].

The choice of the grid transfer operator is crucial for the definition of an effective multigrid method and becomes cumbersome especially for high order problems or on complex domains. Several algebraic multigrid methods have been designed to overcome these difficulties [30, 31, 35]. Simple geometric grid transfer operators are of interest for both geometric and algebraic multigrid methods due to their simplicity and applicability for re-discretizations of the problem at coarser levels. The common choice for the grid transfer operators are interpolation operators [34]. We show that a variety of new interpolating and approximating schemes, developed to design curves and surfaces via subdivision, can be successfully used as grid transfer operators for multigrid methods.

The main contribution of this paper are the sufficient conditions on the symbol of a univariate subdivision scheme of general arity that guarantee that the corresponding grid transfer operator leads to an optimal multigrid method. A stationary iterative method is called optimal whenever its convergence rate is linear and the computational cost of each iteration is proportional to the cost of a matrix vector product. To the best of our knowledge, a hint on the possible link between multigrid methods and subdivision schemes can be only found in [36]. However, [36] only presents a special multigrid method with a structure similar to cascadic multigrid [6] without any theoretical analysis.

To clarify the link between multigrid and subdivision, we start by recalling that local Fourier analysis (LFA) [8] is a classical tool for the convergence analysis of multigrid methods with applications in partial differential equations