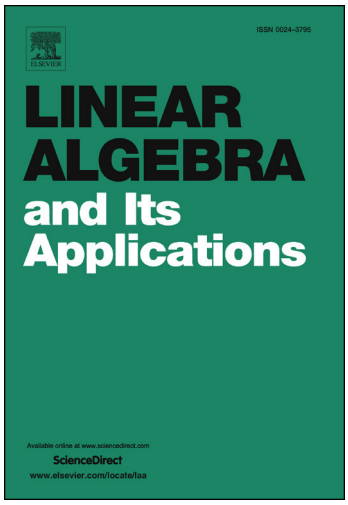

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Maps on states preserving generalized entropy of convex combinations

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Abstract

Let $S(H)$ be the set of all linear positive-semidefinite self-adjoint Trace-one operators (states) on H where H is an at least two-dimensional finite-dimensional real or complex Hilbert space or at least three-dimensional left quaternionic Hilbert space of dimension n. Given a strictly convex function $f : [0,1] \mapsto \mathbb{R}$, for any $\rho \in S(H)$ we define $F(\rho) = \sum_i f(\lambda_i)$, where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of ρ counted with multiplicities. In this note, we completely describe maps $\phi : S(H) \to S(H)$ having the property $F(t\rho + (1-t)\sigma) = F(t\phi(\rho) + (1-t)\phi(\sigma))$ for all $t \in [0,1]$ and every $\rho, \sigma \in S(H)$. It turns out that $\phi(\rho) = U\rho U^*$, $\rho \in S(H)$, where U is a real-linear isometry of H. Note that there is no surjectivity assumption and that our result in particular improves the description of maps preserving the von Neumann entropy of convex combinations of states in the complex Hilbert

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