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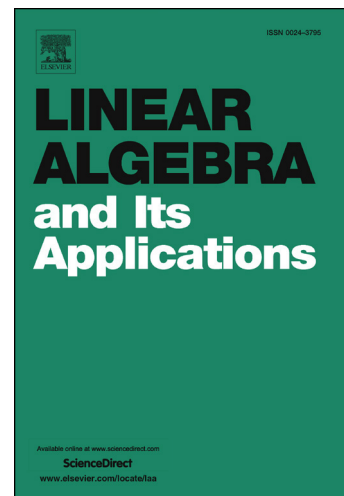
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MORE ON OPERATOR MONOTONE AND OPERATOR CONVEX FUNCTIONS OF SEVERAL VARIABLES

HAMED NAJAFI

ABSTRACT. Let C_1, C_2, \dots, C_k be positive matrices in M_n and f be a continuous real-valued function on $[0, \infty)$. In addition, consider Φ as a positive linear functional on M_n and define

$$\phi(t_1, t_2, t_3, \dots, t_k) = \Phi(f(t_1 C_1 + t_2 C_2 + t_3 C_3 + \dots + t_k C_k)),$$

as a k variables continuous function on $[0, \infty) \times \dots \times [0, \infty)$. In this paper, we show that if f is an operator convex function of order mn , then ϕ is a k variables operator convex function of order (n_1, \dots, n_k) such that $m = n_1 n_2 \dots n_k$. Also, if f is an operator monotone function of order n^{k+1} , then ϕ is a k variables operator monotone function of order n . In particular, if f is a non-negative operator decreasing function on $[0, \infty)$, then the function $t \rightarrow \Phi(f(A + tB))$ is an operator decreasing and can be written as a Laplace transform of a positive measure.

1. INTRODUCTION

Let $\mathbb{B}(\mathcal{H})$ denote the C^* -algebra of all bounded linear operators on a complex Hilbert space $(\mathcal{H}, \langle \cdot, \cdot \rangle)$ and let I be the identity operator. An operator $A \in \mathbb{B}(\mathcal{H})$ is called *positive* if $\langle Ax, x \rangle \geq 0$ holds for every $x \in \mathcal{H}$ and then we can write $A \geq 0$. We say, $A \leq B$ if $B - A \geq 0$; see [1] for other possible orders.

For a continuous real-valued function f and a self adjoint operator A with spectrum in the domain of f , the operator $f(A)$ is defined by the continuous functional calculus. In particular, if \mathcal{H} is a Hilbert space of finite dimension n and $A \in M_n (= \mathbb{B}(\mathcal{H}))$ has the spectral decomposition $A = \sum_{i=1}^n \lambda_i P_i$, where P_i is the projection corresponding to the eigenspace of the eigenvalue λ_i of A , then

$$f(A) = \sum_{i=1}^n f(\lambda_i) P_i.$$

A continuous function $f : J \rightarrow \mathbb{R}$ defined on an interval J is said to be matrix monotone (or matrix increasing) of order n if $A \leq B$ implies that $f(A) \leq f(B)$ for any pair of self adjoint $n \times n$ matrices A, B with spectra in J . A function f is called matrix decreasing of order n if $-f$ is a matrix monotone function of order n . Also,

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