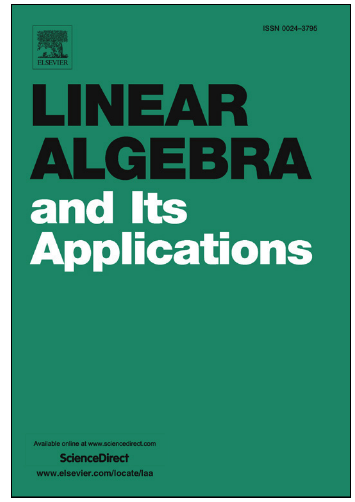

More on operator monotone and operator convex functions of several variables

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MORE ON OPERATOR MONOTONE AND OPERATOR CONVEX FUNCTIONS OF SEVERAL VARIABLES

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ABSTRACT. Let $C_1, C_2, \ldots, C_k$ be positive matrices in M_n and f be a continuous real-valued function on $[0, \infty)$. In addition, consider Φ as a positive linear functional on M_n and define

$$\phi(t_1, t_2, t_3, \ldots, t_k) = \Phi\left(f(t_1C_1 + t_2C_2 + t_3C_3 + \ldots + t_kC_k)\right),$$

as a k variables continuous function on $[0, \infty) \times \ldots \times [0, \infty)$. In this paper, we show that if f is an operator convex function of order mn, then ϕ is a k variables operator convex function of order $(n_1, \ldots, n_k)$ such that $m = n_1n_2 \ldots n_k$. Also, if f is an operator monotone function of order n^{k+1}, then ϕ is a k variables operator monotone function of order n. In particular, if f is a non-negative operator decreasing function on $[0, \infty)$, then the function $t \to \Phi\left(f(A + tB)\right)$ is an operator decreasing and can be written as a Laplace transform of a positive measure.

1. Introduction

Let $\mathbb{B}(\mathscr{H})$ denote the C^*-algebra of all bounded linear operators on a complex Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle)$ and let I be the identity operator. An operator $A \in \mathbb{B}(\mathscr{H})$ is called *positive* if $\langle Ax, x\rangle \geq 0$ holds for every $x \in \mathscr{H}$ and then we can write $A \geq 0$. We say, $A \leq B$ if $B - A \geq 0$; see [1] for other possible orders.

For a continuous real-valued function f and a self adjoint operator A with spectrum in the domain of f, the operator $f(A)$ is defined by the continuous functional calculus. In particular, if $\mathscr{H}$ is a Hilbert space of finite dimension n and $A \in M_n(= \mathbb{B}(\mathscr{H}))$ has the spectral decomposition $A = \sum_{i=1}^{n} \lambda_i P_i$, where P_i is the projection corresponding to the eigenspace of the eigenvalue λ_i of A, then

$$f(A) = \sum_{i=1}^{n} f(\lambda_i)P_i.$$

A continuous function $f : J \to \mathbb{R}$ defined on an interval J is said to be matrix monotone (or matrix increasing) of order n if $A \leq B$ implies that $f(A) \leq f(B)$ for any pair of self adjoint $n \times n$ matrices A, B with spectra in J. A function f is called matrix decreasing of order n if $-f$ is a matrix monotone function of order n. Also,

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