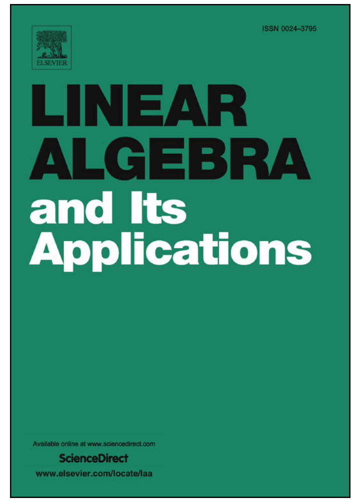

Duality and geodesics for probabilistic frames

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Duality and Geodesics for Probabilistic Frames

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Abstract

Probabilistic frames are a generalization of finite frames into the Wasserstein space of probability measures with finite second moment. We introduce new probabilistic definitions of duality, analysis, and synthesis and investigate their properties. In particular, we formulate a theory of transport duals for probabilistic frames and prove certain properties of this class. We also investigate paths of probabilistic frames, identifying conditions under which geodesic paths between two such measures are themselves probabilistic frames. In the discrete case this is related to ranks of convex combinations of matrices, while in the continuous case this is related to the continuity of the optimal transport plan.

Keywords: `frames, probabilistic frames, optimal transport, Wasserstein metric, duality`
2010 MSC: 42C15, 60D05, 94A12

1. Introduction

1.1. Probabilistic frames in the Wasserstein metric

Frames are redundant spanning sets of vectors or functions that can be used to represent signals in a faithful but nonunique way and that provide an intuitive framework for describing and solving problems in coding theory and sparse representation. We refer to [5, 4, 19] for more details on frames and their applications. To set the notations for this paper, we recall that a set of column vectors $\Phi = \{\varphi_i\}_{i=1}^N \subset \mathbb{R}^d$ is a frame if and only if there exist $0 < A \leqslant B < \infty$ such that

$$\forall x \in \mathbb{R}^d, \quad A\|x\|^2 \leqslant \sum_{i=1}^N \langle x\ , \varphi_i \rangle^2 \leqslant B\|x\|^2.$$

Throughout this paper we abuse notation by also using Φ to denote $[\varphi_1 \ldots \varphi_N]^\top$, the analysis operator of the frame. The (optimal) bounds in the above inequality are the smallest and largest eigenvalues of the frame operator $S_\Phi = \Phi^\top \Phi$.

Continuous frames are natural generalization of frames and were introduced by Ali, Antoine, and Gazeau [1] (see also, [11]). Specifically, let X be a metrizable, locally compact space and ν be positive, inner regular Borel measure for X supported on all of X. Let H be a Hilbert space. Then a set of vectors $\{\eta_x^i, i \in \{1, \cdots, n\}, x \in X\} \subset H$ is a rank-n (continuous) frame if, for each $x \in X$, the vectors $\{\eta_x^i, i \in \{1, \cdots, n\}\}$

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