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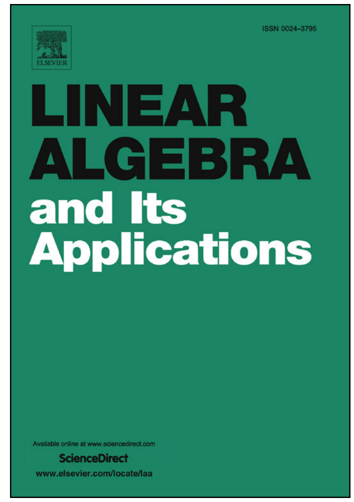

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MAXIMAL DOUBLY STOCHASTIC MATRIX CENTRALIZERS

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ABSTRACT. We describe doubly stochastic matrices with maximal centralizers.

Keywords: Doubly stochastic matrices, Matrix algebra, Centralizers

AMS Subject of Classification: 15A27

1. INTRODUCTION

Let M_n be the algebra of all n-by-n matrices over the complex field $\mathbb{C}$ and let $I \in M_n$ be its identity. One of the relations which is often used on M_n, both in pure and applied problems, is commutativity [13, 14, 15, 16]. In the study of commutativity the notion of *centralizer* (also called commutant) has an important role. For $A \in M_n$ its centralizer, denoted by $\mathcal{C}(A)$, is the set of all matrices commuting with A, that is

$$\mathcal{C}(A) = \{X \in M_n : AX = XA\},$$

and for a set $S \subseteq M_n$ its centralizer, denoted by $\mathcal{C}(S)$, is the intersection of centralizers of all its elements, that is

$$\mathcal{C}(S) = \{X \in M_n : AX = XA, \text{ for every } A \in S\}.$$

The Centralizer Theorem (see [6], p. 113 Corollary 1, or [16], p. 106 theorem 2) states that $\mathcal{C}(\mathcal{C}(A)) = \mathbb{C}[A]$ where $\mathbb{C}[X] \subseteq M_n$ denotes the unital algebra spanned by $X \in M_n$. It is well known that the central elements of M_n are the scalar matrices, $\mathcal{C}(M_n) = \{\alpha I : \alpha \in \mathbb{C}\}$.

The centralizer induces an equivalence relation, $\sim$, and a preorder relation, $\preceq$, on M_n:

- A and B are $\mathcal{C}$-equivalent, $A \sim B$, if $\mathcal{C}(A) = \mathcal{C}(B)$,
- $A \preceq B$ if $\mathcal{C}(A) \subseteq \mathcal{C}(B)$.

For a preoder $\preceq$ on M_n we say that:

- a non-scalar matrix A is minimal if for every matrix X with $\mathcal{C}(X) \subseteq \mathcal{C}(A)$ it follows $\mathcal{C}(A) = \mathcal{C}(X)$,
- a non-scalar matrix A is maximal if for every non-scalar matrix X with $\mathcal{C}(X) \supseteq \mathcal{C}(A)$ it follows $\mathcal{C}(A) = \mathcal{C}(X)$.

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