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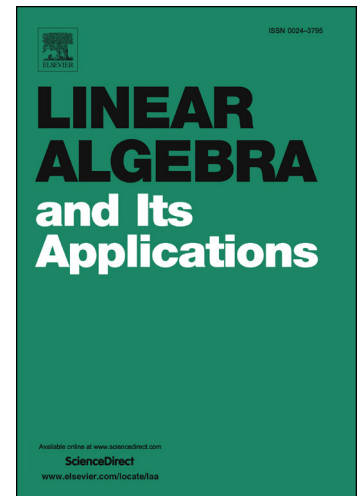
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## MAXIMAL DOUBLY STOCHASTIC MATRIX CENTRALIZERS

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ABSTRACT. We describe doubly stochastic matrices with maximal centralizers.

*Keywords:* Doubly stochastic matrices, Matrix algebra, Centralizers

*AMS Subject of Classification:* 15A27

### 1. INTRODUCTION

Let  $M_n$  be the algebra of all  $n$ -by- $n$  matrices over the complex field  $\mathbb{C}$  and let  $I \in M_n$  be its identity. One of the relations which is often used on  $M_n$ , both in pure and applied problems, is commutativity [13, 14, 15, 16]. In the study of commutativity the notion of *centralizer* (also called commutant) has an important role. For  $A \in M_n$  its centralizer, denoted by  $\mathcal{C}(A)$ , is the set of all matrices commuting with  $A$ , that is

$$\mathcal{C}(A) = \{X \in M_n : AX = XA\},$$

and for a set  $S \subseteq M_n$  its centralizer, denoted by  $\mathcal{C}(S)$ , is the intersection of centralizers of all its elements, that is

$$\mathcal{C}(S) = \{X \in M_n : AX = XA, \text{ for every } A \in S\}.$$

The Centralizer Theorem (see [6], p. 113 Corollary 1, or [16], p. 106 theorem 2) states that  $\mathcal{C}(\mathcal{C}(A)) = \mathbb{C}[A]$  where  $\mathbb{C}[X] \subseteq M_n$  denotes the unital algebra spanned by  $X \in M_n$ . It is well known that the central elements of  $M_n$  are the scalar matrices,  $\mathcal{C}(M_n) = \{\alpha I : \alpha \in \mathbb{C}\}$ .

The centralizer induces an equivalence relation,  $\sim$ , and a preorder relation,  $\preceq$ , on  $M_n$ :

- $A$  and  $B$  are  $\mathcal{C}$ -equivalent,  $A \sim B$ , if  $\mathcal{C}(A) = \mathcal{C}(B)$ ,
- $A \preceq B$  if  $\mathcal{C}(A) \subseteq \mathcal{C}(B)$ .

For a preorder  $\preceq$  on  $M_n$  we say that:

- a non-scalar matrix  $A$  is minimal if for every matrix  $X$  with  $\mathcal{C}(X) \subseteq \mathcal{C}(A)$  it follows  $\mathcal{C}(A) = \mathcal{C}(X)$ ,
- a non-scalar matrix  $A$  is maximal if for every non-scalar matrix  $X$  with  $\mathcal{C}(X) \supseteq \mathcal{C}(A)$  it follows  $\mathcal{C}(A) = \mathcal{C}(X)$ .

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