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Minimal determinantal representations of bivariate polynomials

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ABSTRACT

For a square-free bivariate polynomial p of degree n we introduce a simple and fast numerical algorithm for the construction of $n \times n$ matrices A, B, and C such that $\det(A+xB+yC) = p(x, y)$. This is the minimal size needed to represent a bivariate polynomial of degree n. Combined with a square-free factorization one can now compute $n \times n$ matrices for any bivariate polynomial of degree n. The existence of such symmetric matrices was established by Dixon in 1902, but, up to now, no simple numerical construction has been found, even if the matrices can be nonsymmetric. Such representations may be used to efficiently numerically solve a system of two bivariate polynomials of small degree via the eigenvalues of a two-parameter eigenvalue problem. The new representation speeds up the computation considerably.

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1. Introduction

Let

$$p(x,y) := \sum_{i=0}^{n} \sum_{j=0}^{n-i} p_{ij}\, x^i\, y^j, \tag{1}$$

where $p_{ij} \in \mathbb{C}$ for all i,j, be a bivariate polynomial of degree n, where we assume that $p_{ij} \neq 0$ for at least one index such that $i+j=n$. We say that matrices $A,B,C \in \mathbb{C}^{m\times m}$, where $m \geq n$, form a *determinantal representation* of order m of the polynomial p if

$$\det(A + xB + yC) = p(x,y). \tag{2}$$

It is known since Dixon's 1902 paper [5] that every bivariate polynomial of degree n admits a determinantal representation with symmetric matrices of order n. However, the construction of such matrices is far from trivial and up to now there have been no efficient numerical algorithms, even if we do not insist on matrices being symmetric. We introduce the first efficient numerical construction of determinantal representations that returns $n\times n$ matrices for a square-free bivariate polynomial of degree n, which, with the exception of the symmetry, agrees with Dixon's result. For non square-free polynomials one can combine it with a square-free factorization to obtain a representation of order n.

Our motivation comes from the following approach for finding roots of systems of bivariate polynomials, proposed by Plestenjak and Hochstenbach in [14]. Suppose that we have a system of two bivariate polynomials

$$\begin{aligned} p(x,y) &:= \sum_{i=0}^{n_1} \sum_{j=0}^{n_1-i} p_{ij}\, x^i\, y^j = 0, \\ q(x,y) &:= \sum_{i=0}^{n_2} \sum_{j=0}^{n_2-i} q_{ij}\, x^i\, y^j = 0. \end{aligned} \tag{3}$$

The idea is to construct matrices A_1, B_1, C_1 of size $m_1 \times m_1$ and matrices A_2, B_2, C_2 of size $m_2 \times m_2$ such that

$$\begin{aligned} \det(A_1 + xB_1 + yC_1) &= p(x,y), \\ \det(A_2 + xB_2 + yC_2) &= q(x,y) \end{aligned} \tag{4}$$

and then numerically solve the equivalent two-parameter eigenvalue problem [1]

$$\begin{aligned} (A_1 + xB_1 + yC_1)\, u_1 &= 0, \\ (A_2 + xB_2 + yC_2)\, u_2 &= 0. \end{aligned} \tag{5}$$

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