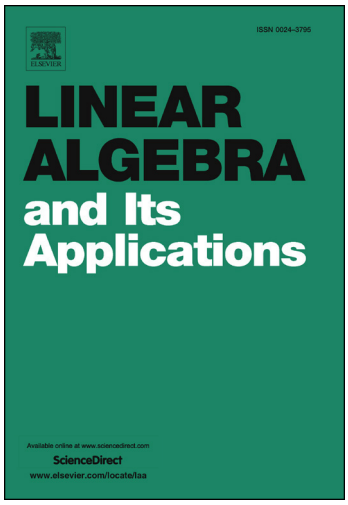

Three observations on spectra of zero-nonzero patterns

Yaroslav Shitov

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Three observations on spectra of zero-nonzero patterns

Yaroslav Shitov

National Research University Higher School of Economics, 20 Myasnitskaya Ulitsa, Moscow 101000, Russia

Abstract

Using standard techniques from combinatorics, model theory, and algebraic geometry, we prove generalized versions of several basic results in the theory of spectrally arbitrary matrix patterns. Also, we point out a counterexample to a conjecture proposed recently by McDonald and Melvin.

Keywords: matrix theory, eigenvalues, zero pattern
2010 MSC: 15A18, 15B35

The study of spectra of zero-nonzero matrix patterns began more than a decade ago (see [1, 2]), and a considerable amount of publications related to matrix completion problems are devoted to this topic. To begin with, recall that an $n \times n$ *zero-nonzero pattern* is a matrix with entries $*$ and 0. In other words, we can think of a zero-nonzero pattern as a class of $n \times n$ matrices which have non-zero elements at the same positions, which are indicated by the $*$ sign. Such a pattern S is called *spectrally arbitrary* with respect to a field $\mathbb{F}$ if any monic polynomial $f \in \mathbb{F}[t]$ of degree n arises as the characteristic polynomial of a matrix with entries in $\mathbb{F}$ and pattern S.

Several well known results on this topic are usually being formulated in the case of the real numbers, and questions often arise as to whether or not the corresponding results are true over other fields. Many results of this kind admit natural generalizations for matrices over arbitrary fields, but some of these generalizations seem to remain unknown for the community. Questions of this kind occasionally appear as *'open problems'* in the literature, so we believe it would be helpful to clarify the situation when possible.

Email address: yaroslav-shitov@yandex.ru (Yaroslav Shitov)