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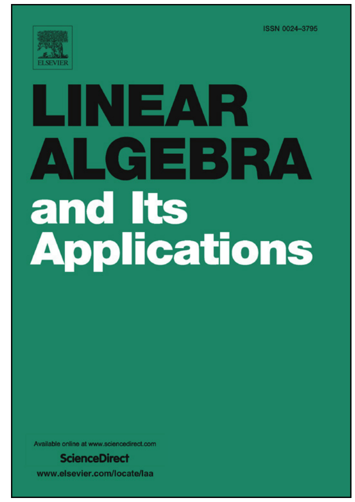

The Hankel matrix rank theorem revisited

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The Hankel matrix rank theorem revisited

Absract. We give a new short proof of a version of a Hankel matrix rank theorem. That version expresses the rank of H by the smallest possible rank of an infinite Hankel matrix containing H. The new approach is based on application of the Kronecker theorem.

Key words: Hankel matrix, rank, submatrix

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A Hankel matrix is a rectangular matrix of form

$$H = \begin{pmatrix} s_1 & s_2 & s_3 & \dots & s_q \\ s_2 & s_3 & s_4 & \dots & s_{q+1} \\ s_3 & s_4 & s_5 & \dots & s_{q+2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ s_p & s_{p+1} & s_{p+2} & \dots & s_l \end{pmatrix} \quad (l = p + q - 1). \tag{1}$$

Thus, a Hankel matrix is characterized by the property that the (i, j) entry depends only on the sum $i + j$. Infinite Hankel matrices

$$H_\infty = \begin{pmatrix} s_1 & s_2 & s_3 & \dots \\ s_2 & s_3 & \cdot & \cdot \\ s_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}. \tag{2}$$

are also considered in the literature. The intense study of Hankel matrices traces back to the second half of the 19th century and it is still continuing due to many applications in algebra, functional analysis, random processes, etc. (see [1-4] and references therein). Algebraic aspects of the theory of Hankel matrices are analysed in the monograph of Iohvidov [2], where the classical results of Frobenius are comprehended and developed. One of the central results of [2] is the theorem on the rank of a complex square Hankel matrix. That theorem is based on the concept of (r, k)-characteristic. Then by a significant complication of this notion the rank theorem was generalized to rectangular matrices [5].

Later [6] we developed a different version of the Hankel matrix rank theorem, where, in contrast to Iohvidov's theorem [5], the key parameter is not the (r, k)-characteristic but the smallest rank of an infinite Hankel matrix containing a given matrix as a corner submatrix. That theorem gave another view on the rank problem of Hankel matrices. Its shortcoming, however, was a quite artificial and technical proof based on some results on generalized Hankel matrices arising in the theory of linear automata [7].

In this note we fill this gap and give a new proof based on a straightforward application of the Kronecker theorem on the basic minor of an infinite Hankel matrix. The new proof is shorter, more natural, and gives a new way to understand the essence of the problem.