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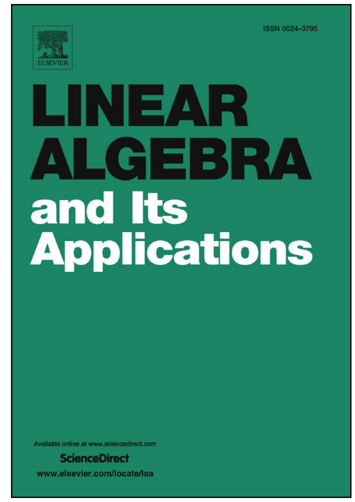

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Two subgraph grafting theorems on the energy of bipartite graphs

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Abstract: The energy of a graph is defined as the sum of the absolute values of all eigenvalues of the graph. The subgraph grafting operation on a graph is a kind of subgraph moving between two vertices of the graph. In this paper, we introduce two new subgraph grafting operations on bipartite graph and show how the graph energy changes under these subgraph grafting operations. As the application of these operations, we determine the trees with the third and fourth minimal energies in the set of trees with given order and domination number.

Key words: Graph; Tree; Energy; Domination number

1 Introduction

Let $G = (V, E)$ be a simple connected graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and edge set E. The *adjacency matrix* of G, $A(G) = (a_{ij})$, is an $n \times n$ matrix, where $a_{ij} = 1$ if v_i and v_j are adjacent and $a_{ij} = 0$, otherwise. Thus $A(G)$ is a real symmetric matrix with zeros on the diagonal, and all eigenvalues of $A(G)$ are real. The *eigenvalues* of graph G are the eigenvalues of $A(G)$, written as $\lambda_1, \lambda_2, \ldots, \lambda_n$. The *energy* of G, denoted by $\mathbb{E}(G)$, is defined [1, 2] as $\mathbb{E}(G) = \sum\limits_{i=1}^{n} |\lambda_i|$.

The *characteristic polynomial* $\det(xI - A(G))$ of the adjacency matrix $A(G)$ of a graph G is also called the characteristic polynomial of G, written as $\Phi(G, x) = \sum\limits_{i=0}^{n} a_i(G)x^{n-i}$.

Let $b_i(G) = |a_i(G)|$, and $\widetilde{\phi}(G, x) = \sum\limits_{i=0}^{n} b_i(G)x^{n-i} = \sum\limits_{i=0}^{n} |a_i(G)|x^{n-i}$. Now if G is a bipartite graph, then $\widetilde{\phi}(G, x)$ has the following form

$$\widetilde{\phi}(G, x) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} b_{2i}(G)x^{n-2i}. \tag{1}$$

For the sake of simplicity, the polynomials $\phi(G, x)$ and $\widetilde{\phi}(G, x)$ will be simply denoted by $\phi(G)$ and $\widetilde{\phi}(G)$, respectively.

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