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Spectral analogues of Moon–Moser's theorem on Hamilton paths in bipartite graphs

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ABSTRACT

In 1962, Erdős proved a theorem on the existence of Hamilton cycles in graphs with given minimum degree and number of edges. Significantly strengthening in case of balanced bipartite graphs, Moon and Moser proved a corresponding theorem in 1963. In this paper we establish several spectral analogues of Moon and Moser's theorem on Hamilton paths in balanced bipartite graphs and nearly balanced bipartite graphs. One main ingredient of our proofs is a structural result of its own interest, involving Hamilton paths in balanced bipartite graphs with given minimum degree and number of edges.

1. Introduction

This is a sequel to our previous paper [12]. In this paper, we are interested in establishing tight spectral sufficient conditions for Hamilton paths in balanced bipartite graphs and nearly balanced bipartite graphs. Throughout this paper, a bipartite graph

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with the bipartition $\{X, Y\}$ is called *balanced* if $|X| = |Y|$; and is called *nearly balanced* if $|X| - |Y| = 1$ (by the symmetry). A graph G is called *Hamiltonian* if it contains a spanning cycle, and is called *traceable* if it contains a spanning path.

The topic of Hamiltonicity of graphs has a long history. In 1961, Ore [24] proved that every graph on n vertices has a Hamilton cycle if $e(G) > \binom{n-1}{2} + 1$. One year later, Erdős [6] generalized Ore's theorem by introducing the minimum degree of a graph as a new parameter. More precisely, Erdős proved that

Theorem 1.1 *(Erdős [6]). Let G be a graph on n vertices, with minimum degree $\delta(G)$. If $n/2 > \delta(G) \geq k \geq 1$, and*

$$e(G) > \max\left\{\binom{n-k}{2} + k^2, \binom{n - \lfloor\frac{n-1}{2}\rfloor}{2} + \left\lfloor\frac{n-1}{2}\right\rfloor^2\right\},$$

then G is Hamiltonian.

Motivated by Erdős' work [6], Moon and Moser [16] presented some corresponding results for balanced bipartite graphs. We state one of their theorems as follows, which is the starting point of our present paper.

Theorem 1.2 *(Moon and Moser [16]). Let G be a balanced bipartite graph on $2n$ vertices, with minimum degree $\delta(G) \geq k$, where $1 \leq k \leq n/2$. If*

$$e(G) > \max\left\{n(n-k) + k^2, n\left(n - \left\lfloor\frac{n}{2}\right\rfloor\right) + \left\lfloor\frac{n}{2}\right\rfloor^2\right\},$$

then G is Hamiltonian.

Compared with the number of edges of graphs, eigenvalues of graphs are also very powerful for describing the structure of graphs. There are several well known examples, such as the spectral proof of Friendship Theorem [7]. For Hamiltonicity of graphs, early pioneer works include those of van den Heuvel [10], Krivelevich and Sudakov [11], Butler and Chung [5], etc.

Recently, spectral extremal graph theory has rapidly developed, where extremal properties of graphs are studied by means of eigenvalues of associated matrices of graphs. In this area, many beautiful and deep results have been proved, such as a spectral Turán theorem [17], a spectral Erdős–Stone–Bollobás theorem [18], a spectral version of Zarankiewicz problem [19], spectral sufficient conditions for paths and cycles [20,25,26], etc. For an excellent survey on recent development of spectral extremal graph theory, we refer the reader to Nikiforov [21]. In particular, on the topic of Hamiltonicity, Fielder and Nikiforov [9] gave spectral analogues of Ore's theorem [24]. More work in this vein can be found in Zhou [27], Lu, Liu and Tian [14], as well as Liu, Shiu and Xue [13]. Towards finding spectral analogues of Erdős' theorem, the first attempt was made by the second