Accepted Manuscript

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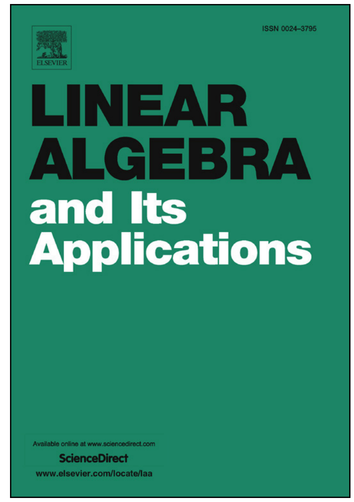

PII: S0024-3795(16)30482-7
DOI: http://dx.doi.org/10.1016/j.laa.2016.10.009
Reference: LAA 13895

To appear in: *Linear Algebra and its Applications*

Received date: 11 January 2016
Accepted date: 10 October 2016

Please cite this article in press as: J. Antezana et al., On some factorizations of operators, *Linear Algebra Appl.* (2016), http://dx.doi.org/10.1016/j.laa.2016.10.009

On some factorizations of operators

Jorge Antezana[a,b,1], M. Laura Arias[a,c,2], Gustavo Corach[a,c,2,*]

[a]Instituto Argentino de Matemática "Alberto P. Calderón", CONICET
Saavedra 15, Piso 3 (1083), Buenos Aires, Argentina.
[b]Dpto. de Matemática, FCE-UNLP, La Plata, Argentina
[c]Dpto. de Matemática, Facultad de Ingeniería, Universidad de Buenos Aires.

Abstract

Given two subsets $\mathcal{A}$ and $\mathcal{B}$ of the algebra of bounded linear operators on a Hilbert space $\mathcal{H}$ we denote by $\mathcal{AB} := \{AB : A \in \mathcal{A}, B \in \mathcal{B}\}$. The goal of this article is to describe $\mathcal{AB}$ if $\mathcal{A}$ and $\mathcal{B}$ denote classes of projections, partial isometries, positive (semidefinite) operators, etc. Moreover, fixed $T \in \mathcal{AB}$ we shall describe $(\mathcal{AB})_T := \{(A, B) \in \mathcal{A} \times \mathcal{B} : AB = T\}$, $p_1((\mathcal{AB})_T) := \{A \in \mathcal{A} : T = AB \text{ for some } B \in \mathcal{B}\}$ and $p_2((\mathcal{AB})_T) := \{B \in \mathcal{B} : T = AB \text{ for some } A \in \mathcal{A}\}$.

Keywords: Factorizations, polar decomposition, projections, partial isometries
2010 MSC: 47A05

1. Introduction

Let $\mathcal{H}$ be a Hilbert space and denote by $\mathcal{L}$ the algebra of bounded linear operators on $\mathcal{H}$. The main goal of this paper is the characterization of

$$\mathcal{AB} = \{AB : A \in \mathcal{A}, B \in \mathcal{B}\}, \tag{1}$$

for certain subsets $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{L}$. Moreover, for $T \in \mathcal{AB}$ we study the set:

$$(\mathcal{AB})_T := \{(A, B) \in \mathcal{A} \times \mathcal{B} : AB = T\},$$

and its natural projections

$$p_1((\mathcal{AB})_T) := \{A \in \mathcal{A} : T = AB \text{ for some } B \in \mathcal{B}\},$$

and

$$p_2((\mathcal{AB})_T) := \{B \in \mathcal{B} : T = AB \text{ for some } A \in \mathcal{A}\}.$$

Of course, it looks impossible to find methods which allow to deal with the problem for general $\mathcal{A}$ and $\mathcal{B}$. We shall show that in many concrete natural cases, the problem is not

*Corresponding author. Address: IAM-CONICET, Saavedra 15, Piso 3 (1083), Buenos Aires, Argentina.
Email addresses: antezana@mate.unlp.edu.ar (Jorge Antezana), lauraarias@conicet.gov.ar (M. Laura Arias), gcorach@fi.uba.ar (Gustavo Corach)
[1]Partially supported by CONICET-PIP 2009-435, UNLP 11X681, MTM2012-36378
[2]Partially supported by PIP 0757(CONICET) and UBACYT 20020130100637