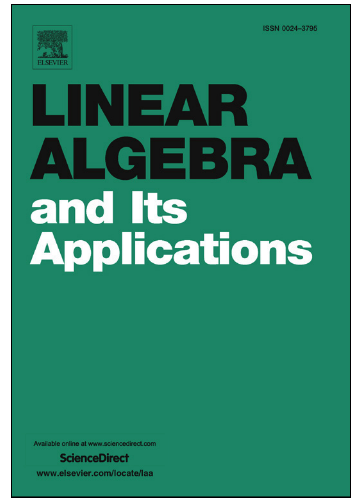

Constructing graphs with given spectrum and the spectral radius at most 2

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Constructing graphs with given spectrum and the spectral radius at most 2

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Abstract

Two graphs are cospectral if their spectra coincide. The set of all graphs that are cospectral to a given graph, including the graph by itself, is the cospectral equivalence class of the graph. We say that a graph is determined by its spectrum, or that it is a DS-graph, if it is a unique graph having that spectrum. Given n reals belonging to the interval $[-2, 2]$, we want to find all graphs on n vertices having these reals as the eigenvalues of the adjacency matrix. Such graphs are called Smith graphs. Our search is based on solving a system of linear Diophantine equations. We present several results on spectral characterizations of Smith graphs.

Key words: Adjacency spectrum, cospectral graphs, DS-graph, cospectral equivalence class, Diophantine equation.

2010 Mathematics Subject Classification: 05C50

1 Introduction

In this paper, we consider only finite undirected simple graphs, i.e. graphs without loops or multiple edges. Let G be a simple graph on n vertices (or a graph of order n), and with the adjacency matrix A. The *characteristic polynomial* $P_G(x) = \det(xI - A)$ of G is the characteristic polynomial of its adjacency matrix A. The eigenvalues of A, in non-increasing order, are denoted by $\lambda_1(G), \ldots, \lambda_n(G)$. They are called the *eigenvalues* of G and they form the *spectrum* of G. The multiplicity k of the eigenvalue λ_i in the spectrum of G will be denoted by $[\lambda_i]^k$. Since A is real and symmetric, the spectrum of G consists of reals. In particular, $\lambda_1(G)$, as the largest eigenvalue of G, is called the *spectral radius* (or *index*) of G.

The spectrum of G (as a multi-set or family of reals) will be denoted by $\widehat{G}$. The *disjoint union* of graphs G_1 and G_2 will be denoted by $G_1 + G_2$, while for the union of their spectra (i.e. the spectrum of $G_1 + G_2$) we will use the following mark $\widehat{G}_1 + \widehat{G}_2$. In the similar manner, kG ($k\widehat{G}$) stands for the union of k copies of G (the spectrum of kG). In this sense, we shall also use linear combinations of graphs and of their spectra.

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